

## Broadcast Ephemeris Model of the BeiDou Navigation Satellite System

Xie Xiaogang\* and Lu Mingquan

Department of Electronic Engineering, Tsinghua University, Beijing 100084, China

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### Abstract

The BeiDou Navigation Satellite System (BDS) space constellation is composed of geostationary earth orbit (GEO), medium earth orbit (MEO), and inclined geosynchronous satellite orbit (IGSO) satellites, all of which adopt the Global Positioning System (GPS) broadcast ephemeris model of the United States of America (U.S.A). However, in the case of small eccentricity and small orbit inclination angle, the coefficient matrix is non-positive when fitting the broadcast ephemeris parameters due to the fuzzy definition of a few Keplerian orbit elements, thereby resulting in low precision or even failure. This study proposed a novel broadcast ephemeris model based on the second class of non-singular orbit elements. The proposed model can address the unsuitability of the classical broadcast ephemeris model in the condition of small eccentricity and small orbit inclination angle, and unify the broadcast ephemeris parameters user algorithm models of GEO, MEO, and IGSO. The proposed model is a 14-parameters broadcast ephemeris model constructed with the second class of non-singular orbit elements and the corresponding broadcast ephemeris parameters perturbation over time. Lastly, the proposed model was verified by precision orbit data generated by the Satellite Tool Kit (STK). Results show that the 14-parameters broadcast ephemeris model is suitable for the GEO, MEO, and IGSO satellites and has high fitting precision to fully meet the requirements of the BDS. Moreover, compared with the existing 16-parameters broadcast ephemeris model, two ephemeris parameters are decreased without reducing precision, thereby possibly saving communication resources. The proposed method provides a good prospect to optimize the design of the BDS broadcast ephemeris model.

*Keywords:* BDS, Broadcast ephemeris, URE (User Range Error), Non-singular orbit elements, Keplerian orbit elements

### 1. Introduction

The broadcast ephemeris parameters are important components of a satellite navigation system. The navigation message, which is the final expression form of satellite precision orbit data given to users, provides high-precision satellite location information to realize navigation and positioning [1]. The simplicity and efficiency of the broadcast ephemeris model directly determine the performance of rapid navigation and positioning [2]. The BeiDou satellite navigation system (BDS) is a hybrid constellation composed of geostationary Earth orbit (GEO), medium Earth orbit (MEO), and inclined geosynchronous satellite orbit (IGSO) satellites. Although the orbit characteristics of these satellites have many differences to one another, the BDS still uses the classic satellite broadcast ephemeris model [3]. This model comprises 16 parameters based on six Keplerian orbit elements, ephemeris reference time, orbit perturbation harmonic coefficient, and its perturbation variables changing with time [4].

The classic broadcast ephemeris model is based on six Keplerian orbit elements, namely, semi-major axis, eccentricity, argument of perigee, mean anomaly, longitude

of the ascending node of the orbit plane and orbit inclination. Each of these elements has a clear physical meaning. When the satellite orbit is near-circular, the orbit eccentricity approaches or equals to 0, the argument of perigee will become meaningless, thereby resulting in the perigee latitude, mean anomaly, eccentric anomaly, and true anomaly measured from the perigee becoming meaningless as well. When the orbital plane coincides with the equatorial plane, the orbital inclination approaches or equals to 0 or 180°, the ascending node of the orbit plane, ascending longitude, and perigee longitude measured from the ascending node of the orbit plane become meaningless [5]. In view of these two cases, the coefficient matrix is non-positive when fitting the classic broadcast ephemeris parameters, thereby resulting in low precision or even failure.

This study established a broadcast ephemeris model by analyzing the second class of non-singular orbit elements. Long-term simulation was performed by adopting different orbit data to verify the proposed model. This method aims to address the adaptability of the existing broadcast ephemeris model and unify the BDS GEO satellite and non-GEO satellite broadcast ephemeris model to improve the efficiency of the engineering design. Accordingly, this provides a good prospect to optimize the design of the BDS broadcast ephemeris model.

\*E-mail address: jasonxx@163.com

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2. State of the art

At present, the operational navigation systems that have been built and are being built include the Global Positioning System (GPS) of the United States of America (U.S.A) [4], the Global Navigation Satellite System (GLONASS) of Russia [6], the BDS of China [3], the Galileo of Europe [7], the Quasi-Zenith Satellite System (QZSS) of Japan [8], and other satellite-based augmentation systems. Because of the inconsistent application, the key points of the design of these navigation systems are different. Thus, the broadcast ephemeris models of these systems are dissimilar. The existing broadcast ephemeris models are classified into two types. The first type adopts satellite position vector, satellite velocity vector and simplified dynamic parameters (e.g., GLONASS [6], satellite-based augmentation systems [8]) as its broadcast ephemeris parameters. Another type adopts the broadcast ephemeris parameters that comprises the Keplerian orbit elements and perturbation variables, such as GPS [4], BDS [3], and Galileo [7], among others.

The first type of broadcast ephemeris model is suitable for any navigation satellite orbit, such as GEO, MEO, and IGSO. However, this model requires complex orbit integral algorithm to obtain real-time satellite position [6]. This requirement increases the complexity of satellite position calculation and reduces the system localization real-time ability. Reid T [9] and Sakai Takeyasu [10] analyzed the GEO satellite broadcast ephemeris form and precision of the satellite-based augmentation system (SBAS). Goeken D analyzed the broadcast ephemeris form and precision of GLONASS [11], the conclusion was that the model had short prediction time and poor precision for satellite orbit. Therefore, the existing satellite navigation system broadcast ephemeris model extensively adopts the second broadcast ephemeris model.

The second class of broadcast ephemeris model has been designed by the U.S.A navigation system for the MEO navigation satellite [6]. This model was verified to be applicable to the IGSO navigation satellite [3]. Jefferson D C [12] and Steigenberger P [13] analyzed the accuracy of the GPS broadcast ephemeris, the conclusion was that the broadcast ephemeris user algorithm model was simple with high prediction ability and precision. However, this model has limited adaptability for satellite orbits with small eccentricity and small inclination angle (e.g., GEO navigation satellite). Xie X [14] and Du L [15] proposed a broadcast ephemeris model based on the second class of non-singular orbit elements and a new 16-parameters GEO broadcast ephemeris model, respectively, to solve the aforementioned broadcast ephemeris model adaptability problem. However, these methods increase the complexity of the navigation user algorithm.

The current research primarily aims to address the limitations of the existing broadcast ephemeris model. However, only a few studies have been conducted on the uniformity of the BDS GEO, MEO, and IGSO satellite broadcast ephemeris models. A 14-parameters broadcast ephemeris user algorithm model based on the second class of non-singular orbit elements was proposed for the uniformity of the BDS GEO, MEO, and IGSO satellite broadcast ephemeris models to address the non-positivity of orbits with small eccentricity and small orbit inclination. The calculations and fitting methods of a 14-parameters broadcast ephemeris are provided in detail. These methods can provide the basis for the unification and optimization of the broadcast ephemeris model of BDS.

The remainder of this study is organized as follows. Section 3 establishes a 14-parameters broadcast ephemeris user algorithm model based on the second class of non-singular orbit elements and provides the fitting method of the model parameters. Section 4 simulates and analyzes the proposed model by adopting small eccentricity and small inclination orbit data. Section 5 presents the conclusions.

3. Methodology

The broadcast ephemeris parameters of the BDS are a set of extended Keplerian orbit elements that are adopted to predict and extrapolate satellite orbit data. The BDS navigation users can resolve the navigation satellite broadcast ephemeris parameters by receiving navigation message. Calculations and extrapolations of the navigation satellite position can be performed based on the broadcast ephemeris parameters user algorithm provided by the navigation system interface control file. Tab.1 shows the BDS broadcast ephemeris parameters at present [3].

Table 1. 16-parameters broadcast ephemeris parameters of the BDS

	Ephemeris Data Definitions
$t_{oe}$	Ephemeris Reference Time
$a$	the Semi-Major Axis
$e$	Eccentricity
$\omega$	Argument of Perigee
$\Delta n$	Mean Motion Difference From Computed Value
$M_0$	Mean Anomaly at Reference Time
$\Omega_0$	Longitude of Ascending Node of Orbit Plane at Weekly Epoch
$\dot{\Omega}$	Rate of Right Ascension
$i$	Inclination Angle at Reference Time
$\dot{i}$	Rate of Inclination Angle
$C_{uc}$	Amplitude of the Cosine Harmonic Correction Term to the Argument of Latitude
$C_{us}$	Amplitude of the Sine Harmonic Correction Term to the Argument of Latitude
$C_{rc}$	Amplitude of the Cosine Harmonic Correction Term to the Orbit Radius
$C_{rs}$	Amplitude of the Sine Harmonic Correction Term to the Orbit Radius
$C_{ic}$	Amplitude of the Cosine Harmonic Correction Term to the Angle of Inclination
$C_{is}$	Amplitude of the Sine Harmonic Correction Term to the Angle of Inclination

The second class of non-singularity orbit elements is introduced based on the classic broadcast ephemeris model to replace the classical six orbit elements. And these variables are considered basic variables of the satellite's orbit. The relationship between the second class of non-singularity orbit elements and the classical six orbit elements is defined as follows [16]:

$$\xi = e \cos \tilde{\omega}, \eta = -e \sin \tilde{\omega} \tag{1}$$

$$h = \sin i \cos \Omega, k = -\sin i \sin \Omega \tag{2}$$

$$\lambda = M + \tilde{\omega}, \tilde{\omega} = \omega + \Omega \tag{3}$$

The aforementioned second class non-singular orbit elements are used as basis to design the 14-parameters navigation satellite broadcast ephemeris (see Tab.2) [17, 18]:

**Table 2.** 14-parameters broadcast ephemeris parameters of the BDS

Ephemeris Data Definitions	
$t_{oe}$	Ephemeris Reference Time
$a$	the Semi-Major Axis
$\xi, \eta$	Two-dimensional Component of Eccentricity
$h, k$	Two-dimensional Component of Inclination Angle
$\lambda$	Mean Longitude at Reference Time
$\dot{\lambda}$	Rate of Mean Longitude
$\dot{h}, \dot{k}$	Rate of Two-dimensional Component of Inclination Angle
$C_{uc}$	Amplitude of the Cosine Harmonic Correction Term to the Argument of Latitude
$C_{us}$	Amplitude of the Sine Harmonic Correction Term to the Argument of Latitude
$C_{rc}$	Amplitude of the Cosine Harmonic Correction Term to the Orbit Radius
$C_{rs}$	Amplitude of the Sine Harmonic Correction Term to the Orbit Radius

**3.1 14-parameters broadcast ephemeris user algorithm**

The calculation steps of the user algorithm model based on 14-parameters broadcast ephemeris are as follows [17,18]:

(1) Calculate the time from ephemeris reference epoch:

$$t_i = t - t_{oe} \tag{4}$$

(2) Calculate the average motion speed:

$$n = \sqrt{\frac{\mu}{a^3}} \tag{5}$$

where  $\mu$  is the earth's gravitational constant.

(3) Calculate the mean longitude:

$$\lambda_i = \lambda + (n + \dot{\lambda}) \cdot t_i \tag{6}$$

(4) Calculate the eccentric longitude by iteration:

$$E_i = \lambda_i + \xi \cdot \sin E_i + \eta \cdot \cos E_i \tag{7}$$

(5) Calculate the radius vector:

$$r_i = a \cdot (1 - \xi \cdot \cos E_i + \eta \cdot \sin E_i) \tag{8}$$

(6) Calculate the true longitude:

$$\sin u_i = \frac{a}{r_i} \cdot \left( \sin E_i + \eta - \xi \cdot \frac{\sqrt{\xi^2 + \eta^2 - (1 - r_i/a)^2}}{1 + \sqrt{1 - \xi^2 - \eta^2}} \right) \tag{9}$$

$$\cos u_i = \frac{a}{r_i} \cdot \left( \cos E_i - \xi - \eta \cdot \frac{\sqrt{\xi^2 + \eta^2 - (1 - r_i/a)^2}}{1 + \sqrt{1 - \xi^2 - \eta^2}} \right) \tag{10}$$

$$u_i = \arctan \left( \frac{\sin u_i}{\cos u_i} \right) \tag{11}$$

(7) Calculate the correctional two-dimensional components of eccentricity:

$$h_i = h + \dot{h} \cdot t_i, k_i = k + \dot{k} \cdot t_i \tag{12}$$

(8) Calculate the correctional orbit radius vector and true longitude:

$$r_i' = r_i + C_{rc} \cdot \cos(2 \cdot u_i) + C_{rs} \cdot \sin(2 \cdot u_i) \tag{13}$$

$$u_i' = u_i + C_{uc} \cdot \cos(2 \cdot u_i) + C_{us} \cdot \sin(2 \cdot u_i) \tag{14}$$

(9) Calculate the satellite position in inertial frame:

$$r_{eci} = r_i' \cdot \cos u_i' \cdot P^* + r_i' \cdot \sin u_i' \cdot Q^* \tag{15}$$

where

$$P^* = \begin{pmatrix} 1 - \frac{k_i^2}{1 + \cos I} & -\frac{h_i \cdot k_i}{1 + \cos I} & k_i \end{pmatrix}^T \tag{16}$$

$$Q^* = \begin{pmatrix} -\frac{h_i \cdot k_i}{1 + \cos I} & 1 - \frac{h_i^2}{1 + \cos I} & h_i \end{pmatrix}^T \tag{17}$$

$$\cos I = \sqrt{1 - h_i^2 - k_i^2} \tag{18}$$

(10) Calculate the satellite position in CGCS2000:

$$r_{ecf} = R_z(\omega_e \cdot t_i) \cdot r_{eci} \tag{19}$$

where  $\omega_e$  is the earth's rotation rate.  $R_z$  is the rotating matrix.

**3.2 14-parameters broadcast ephemeris fitting algorithm**

The broadcast ephemeris fitting model constructed based on the 14-parameters broadcast ephemeris user algorithm model in Section 3.1 is as follows:

$$\text{State equation: } X = X(X_0, t_0, t) \tag{20}$$

$$\text{Observation equation: } Y = Y(X, t) = Y(X_0, t_0, t) \tag{21}$$

where  $X_0$  represents the 14-parameters broadcast ephemeris of BDS at time  $t_{oe}$  and  $Y$  represents the  $m(m \geq 13)$  observation column vector and one of the observation corresponds to one satellite position vector at time  $t$ . The state and observation equations established based on the broadcast ephemeris user algorithm are nonlinear equations. The fitting process of broadcast ephemeris parameters is a least squares estimation problem of the nonlinear system. The nonlinear equations should be linearized and underwent iterations to obtain the solution [19].

Order  $X_{i/0}$  represents the initial value of the broadcast ephemeris at the  $i^{th}$  iteration. If the equation is expanded at

$X_{i/0}$ , and the small high-order term is ignored, then the linear error equation obtained is as follows:

$$Y = Y(X_{i/0}, t_0, t_i) + \left( \frac{\partial Y}{\partial X_0} \right)_{X_0=X_{i/0}} \cdot (X_0 - X_{i/0}) + O\left[ (X_0 - X_{i/0})^2 \right] \quad (22)$$

Ignore the second-order term in Equation (22):

$$y = H \cdot x_0 + v \quad (23)$$

Where

$$x_0 = X_0 - X_{i/0} \quad (24)$$

$$y = Y - Y(X_{i/0}, t_0, t) \quad (25)$$

$$H = \left( \frac{\partial Y}{\partial X_0} \right)_{X_0=X_{i/0}} = \left( \frac{\partial Y}{\partial X} \cdot \frac{\partial X}{\partial X_0} \right)_{X_0=X_{i/0}} \quad (26)$$

The optimal value of  $x_0$  is obtained based on the least square principle:

$$\hat{x}_0 = (H^T H)^{-1} H^T y \quad (27)$$

Thereafter, the estimation parameters after the  $i^{th}$  number of iterations is as follows:

$$X_{(i+1)/0} = X_{i/0} + \hat{x}_0 \quad (28)$$

The iteration ending condition is as follows:

$$\begin{cases} |\sigma_{i+1}| < \delta_1, |\sigma_{i+1} - \sigma_i| < \delta_2 \\ N < N_{\max} \end{cases} \quad (29)$$

where

$$\sigma_i = \sqrt{v^T v / (m - 13)} \quad (30)$$

where  $\delta_1$  and  $\delta_2$  are the arbitrary small quantities that are set based on the broadcast ephemeris fitting precision (normally  $\delta_1 = 10^{-6}$ ,  $\delta_2 = 10^{-2}$ ), in which iteration times  $N_{\max}$  represents the max iteration times (typically  $N_{\max} = 30 \sim 50$ ). The  $\sigma_i$  represents the unit variance of particle weight at  $i^{th}$  number of iteration.

The calculation formulas of the partial derivative of the non-singular orbit elements in the measurement matrix and the transfer matrix are derived and provided [20]. The calculation method of other partial derivative of broadcast ephemeris parameters is discussed in reference [19].

Measurement matrix  $\partial Y / \partial X$ :

$$\frac{\partial r}{\partial \xi} = A \cdot r + B \cdot \dot{r}, \frac{\partial r}{\partial \eta} = C \cdot r + D \cdot \dot{r} \quad (31)$$

$$\frac{\partial r}{\partial h} = H \times \dot{r}, \frac{\partial r}{\partial k} = K \times r \quad (32)$$

$$\frac{\partial r}{\partial a} = \frac{r}{a}, \frac{\partial r}{\partial \lambda} = \frac{\dot{r}}{n} \quad (33)$$

$$\frac{\partial r}{\partial \lambda \dot{\theta}} = \frac{\partial r}{\partial k \dot{\theta}} = \frac{\partial r}{\partial h \dot{\theta}} = 0 \quad (34)$$

$$\begin{cases} \frac{\partial r}{\partial C_{rc}} = \cos(2 \cdot u) \cdot (P^* \cdot \cos u + Q^* \cdot \sin u) \\ \frac{\partial r}{\partial C_{rs}} = \sin(2 \cdot u) \cdot (P^* \cdot \cos u + Q^* \cdot \sin u) \end{cases} \quad (35)$$

$$\begin{cases} \frac{\partial r}{\partial C_{uc}} = r \cdot \cos(2 \cdot u) \cdot (-P^* \cdot \sin u + Q^* \cdot \cos u) \\ \frac{\partial r}{\partial C_{us}} = r \cdot \sin(2 \cdot u) \cdot (-P^* \cdot \sin u + Q^* \cdot \cos u) \end{cases} \quad (36)$$

where the calculation of  $A, B, C, D, H, K$  are included in references [18] and [21].

State transfer matrix  $\partial X / \partial X_0$ :

$$\frac{\partial \lambda}{\partial a} = -\frac{3}{2} \sqrt{\frac{\mu}{a^3}} \frac{\partial \lambda}{\partial \lambda \dot{\theta}} = \frac{\partial h}{\partial h \dot{\theta}} = \frac{\partial k}{\partial k \dot{\theta}} = t_i \quad (37)$$

#### 4. Result analysis and discussion

This section substantially analyzes the feasibility and precision of the 14-parameters broadcast ephemeris model. Moreover, the adaptability of 14-parameters broadcast ephemeris model to the MEO, IGSO, and GEO satellite is discussed.

**Table 3.** Initial orbit elements of the satellites

	$a$ (km)	$e$	$i$ (deg)	$\Omega$ (deg)	$\omega$ (deg)	$M$ (deg)
MEO	25506.5	0.0	55.0	120	0.0	0.0
IGSO	42164.0	0.0	55.0	120	0.0	0.0
GEO	42164.0	0.0	0.0	120	0.0	0.0

The satellite precision orbit data to fit the broadcast ephemeris parameters was generated by the Satellite Tool Kit (STK) (10.0.0 version) software during the simulation process. The perturbation factors of the  $21 \times 21$  order non-spherical gravitational perturbation of the Earth, solar perturbation based on the JPL ephemeris, solid tidal perturbation, and atmospheric resistance perturbation were mainly considered [22,23]. Tab.3 shows the initial Keplerian orbits of the MEO, IGSO, and GEO satellites and the initial time was 1 Jan 2010 00: 00: 00.000 UTCG.

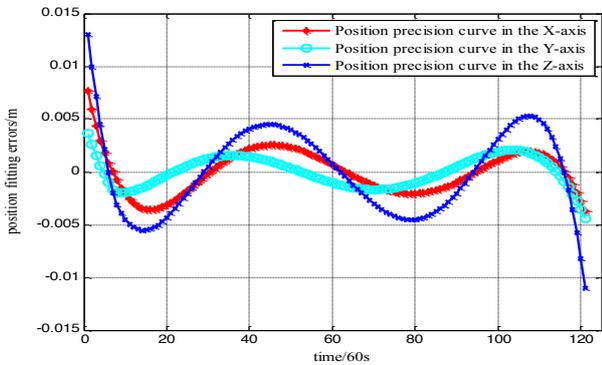
##### 4.1 Precision analysis of the 14-parameters broadcast ephemeris model

The 14-parameters broadcast ephemeris of the MEO, IGSO, and GEO satellites was fitted based on the STK precision orbit data. The length of the fitting arc was two-hours and the sampling interval of the orbital data was one minute.

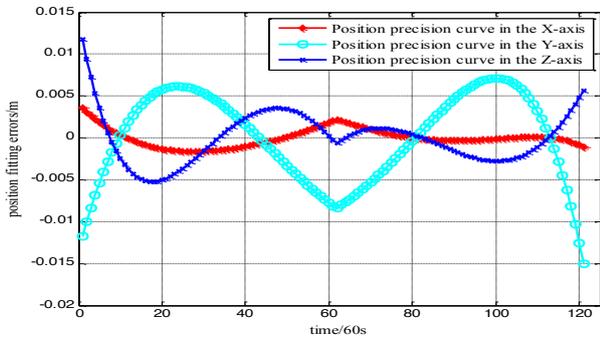
Tab.4 shows the results of the broadcast ephemeris parameters of the three satellites.

**Table 4.** The fitted broadcast ephemeris parameters of the MEO, IGSO, and GEO satellites

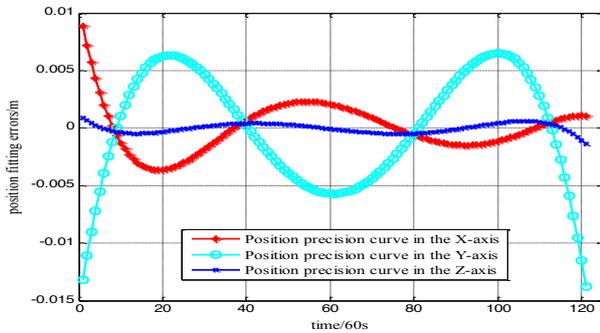
Estimated parameters	MEO	IGSO	GEO
$t_{oe}$	3600.00	3600.00	3600.00
$a$	25504603.56482116	42161505.36507898	42161516.67322837
$\zeta$	-6.75473607e-005	-7.77585965e-005	-8.16065081e-005
$\eta$	9.33338276e-006	2.79163103e-005	-2.43273784e-005
$\lambda$	0.637130977205	0.341648548555	0.3419792836020
$h$	0.816025475519	0.816043147778	-0.000913480293
$k$	-0.065180632982	-0.065204283258	-0.0004181623534
$\dot{\lambda}$	-3.96536475e-009	-6.96736108e-010	1.01261899e-009
$\dot{h}$	9.44818261e-010	8.75396069e-010	1.25968959e-009
$\dot{k}$	7.23181023e-009	1.05482895e-009	3.80192677e-010
$C_{rc}$	166.23176746311	-783.6722122451	-956.6248377799
$C_{rs}$	-24.77757696828	-587.9840488807	502.72684643396
$C_{uc}$	-4.69205283e-006	-2.00868086e-005	1.62840796e-005
$C_{us}$	1.20768340e-005	3.29276358e-005	3.12681104e-005



**Fig. 1.** Position precision curve of the MEO broadcast ephemeris



**Fig. 2.** Position precision curve of the IGSO broadcast ephemeris



**Fig. 3.** Position precision curve of the GEO broadcast ephemeris

Figs. 1, 2, and 3 show the precision curves of the satellite broadcast ephemeris model in the two-hours fitting arc of the MEO, IGSO, and GEO satellites, respectively. The abscissa is the sampling point of the satellite precision orbit, while

the ordinate is the position prediction error of the broadcast ephemeris in the X-axis, Y-axis, and Z-axis directions. Tab.5 provides the absolute values of the satellite position prediction precision of the MEO, IGSO, and GEO 14-parameters broadcast ephemeris in two-hours fitting arc.

Figs. 1, 2, and 3 show that the maximum position fitting errors of the 14-parameters broadcast ephemeris fitting model of the MEO, IGSO, and GEO satellites are 0.0130 m, 0.0151 m, and 0.0138 m, in the X-axis, Y-axis, and Z-axis directions respectively. The maximum error is below 1.6 cm. The mean values of the position fitting error in the X-axis, Y-axis, and Z-axis directions of the MEO, IGSO, and GEO satellites are expressed in millimeter. Thus, the 14-parameters broadcast ephemeris of MEO, IGSO, and GEO can be correctly fitted, and the position prediction precision of MEO, IGSO, and GEO is high within the two-hours effective period of the broadcast ephemeris.

**Tab.5.** Position prediction precision of the MEO, IGSO, and GEO broadcast ephemeris

		X-axis/m	Y-axis/m	Z-axis/m
MEO	Mean	0.0017	0.0012	0.0034
IGSO	absolute	8.197e-4	0.0047	0.0023
GEO	value	0.0016	0.0043	3.399e-4

The analysis indicates that the 14-parameters broadcast ephemeris fitting model is suitable for the MEO, IGSO, and GEO navigation satellites. The position fitting precision is expressed in centimeter, even millimeter, thereby completely satisfying the broadcast ephemeris position forecast precision requirements for the satellite navigation positioning [24,25].

**4.2 Long-term forecast adaptability analysis of the 14-parameters broadcast ephemeris model**

The user range error (URE) of the constellation was adopted to evaluate the forecast precision of the broadcast ephemeris, which was calculated as follows [26]:

$$URE = \sqrt{R_{err}^2 + 0.0192 \cdot (T_{err}^2 + N_{err}^2)} \quad (38)$$

where  $R_{err}$  represents the radial error,  $T_{err}$  represents the along track error, and  $N_{err}$  represents the normal error.

The initial satellite orbit elements are presented in Tab.1. The 7-days precision orbit data of the MEO, IGSO, and GEO satellites were generated by STK. Moreover, the 14-parameters broadcast ephemeris of these satellites was fitted. The length of the fitting arc was two-hours. The sampling interval of the orbital data was one minute and 167 sets of 14-parameters broadcast ephemeris were fitted.

The 167 sets of fitted broadcast ephemeris parameters were used as bases to forecast the satellite position of the MEO, IGSO, and GEO satellites by adopting the 14-parameters broadcast ephemeris user algorithm model in Section 3.1. Figs. 4, 5, and 6 show the URE curves of the MEO, IGSO, and GEO satellites caused by the broadcast ephemeris fitting error within one week.

Figs. 4, 5, and 6 show that the maximum URE caused by the fitted 14-parameters broadcast ephemeris of MEO, IGSO, and GEO are 0.0342 m, 0.0117 m, and 0.0108 m, respectively. The average URE caused by the broadcast ephemeris in one week are 0.0032 m, 0.0012 m, and 0.0014 m, respectively. Thus, the long-term forecast precision of 14-parameters broadcast ephemeris model for the three kinds of satellites is in centimeter order. Meanwhile, the precision of IGSO and GEO are approximately three times higher than that of MEO. The trend of MEO and IGSO are consistent. In the first half of the week, the precision is relatively low and gradually increases toward the second half of the week. The IGSO has the best accuracy in the second half of the week at a maximum of 0.004 m. The prediction precision of the GEO satellite is relatively stable throughout the week.

The 14-parameters broadcast ephemeris model exhibits high precision in position forecasting of the MEO, IGSO, and GEO satellites within one week. Therefore, the model can fully meet the long-term prediction precision requirement of the navigation systems of the MEO, IGSO, and GEO satellites.

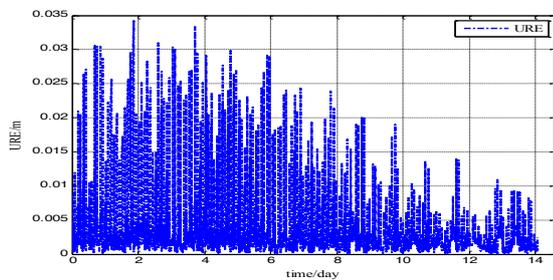


Fig. 4. URE precision curve of the MEO broadcast ephemeris within seven days

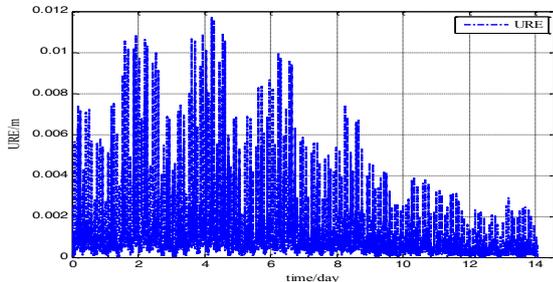


Fig.5. URE precision curve of IGSO the broadcast ephemeris within seven day

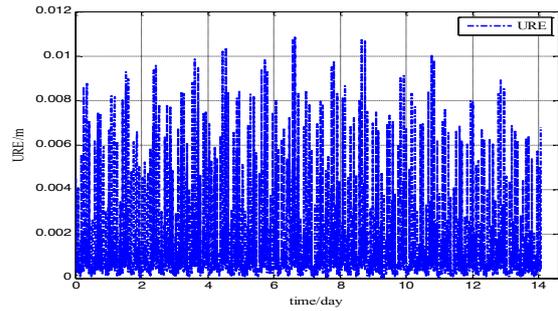


Fig.6. URE precision curve of the GEO broadcast ephemeris within seven days

## 5 Conclusions

This study proposed a 14-parameters broadcast ephemeris user algorithm model by analyzing the existing broadcast ephemeris parameters model to unify the design of the MEO, IGSO, and GEO satellites. And a 14-parameters broadcast ephemeris fitting method was also provided. The relevant formulas were derived and verified using the precision orbit data generated by STK. The conclusions of this study are as follows.

(1) The 14-parameters broadcast ephemeris model is adaptable for the GEO, MEO, and IGSO satellites of BDS. The broadcast ephemeris user algorithm model and broadcast ephemeris parameters fitting model of BDS can be unified.

(2) The satellite orbit fitting precision of the 14-parameters broadcast ephemeris model for the GEO, MEO, and IGSO satellites of BDS are high and completely meet the satellite position forecast requirements of BDS.

(3) Compared with the existing 16-parameters classic broadcast ephemeris model, two ephemeris parameters are decreased without reducing the precision, thereby possibly saving communication resources.

This study proposed a 14-parameters broadcast ephemeris model to redesign the BDS broadcast ephemeris. The proposed model addresses the limited adaptability of the existing broadcast ephemeris model and unifies the broadcast ephemeris user algorithm model of the GEO, MEO, and IGSO satellites of BDS without reducing precision. This study has important guiding significance to the broadcast ephemeris model design of the GEO, MEO, and IGSO hybrid navigation constellation. In future research, the performance of the proposed model should be analyzed based on actual engineering data due to the lack of current actual measurement satellite orbit data.

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