

Research Article

Optimal Supervisory Control of Two Phases Intersection Using Hybrid-based Model Approach**Bilal Tolbi***Department of Automatic, University of Sidi Bel-Abbes, BP 89, 22000, Sidi Bel-Abbes, Algeria.
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Abstract

In the present paper, a hybrid approach that combines modular modelling and Timed Petri Nets model is proposed, the main interest of this combination is to ensure optimal supervisory control of urban traffic flows in two phases intersections of medium-sized cities. Each intersection is considered as a multi-agent system, composed of all agents who represent all physical spaces. The behaviours of interconnected agents are supervised by the Timed Petri Nets model that controls the traffic light cycles according to an optimal timing plan defined by the Continuous Genetic Algorithm. A real two phases intersection in Sidi Bel-Abbes City is presented as an example. The proposed timing plan is compared with the current one using SUMO software for the most important objective functions. Also, the proposed model's behaviour is compared with TSIS-CORSIM software's model, and all obtained experiments results show the effectiveness of the approach and its possibility to be extended and applied to a many-phases intersection.

Keywords: CGA, MAS-based model, timed Petri net, supervisory control, timing plan.

1. Introduction

The rapid growth in transportation demand within the limited capacity in infrastructures, Urban Traffic Network (UTN) has become one of significant socio-economic issues in urban city life. Especially during last decades, the traffic congestion phenomenon has become predominant and drew great attention due to the world energy crisis and environmental worries. Different classical strategies try to reduce this phenomenon by the construction of new infrastructures, widening of busy roads, encouraging the use of public transport and implementing transport strategies (e.g. optimize the use of existing infrastructure). However, these solutions are still far for facilitate an optimal managing of traffic because they are unable to reply to the flow fluctuation in demand. In this purpose, the best solution for an optimal managing of the traffic congestion will be sought in the signal control strategy.

Recently, some researches propose a new view, the UTN is considered as Hybrid Dynamical System (HDS) [1]. In the case where the UTN is large, cycle length of system-wide becomes unsuitable by taking the network as a single system. Indeed, there are some researchers interested to divide UTN into single intersections and study each intersection separately see eg [2]. When the number of intersections increases, more variables need to be determined and more constraints are imposed. Therefore the analytical control approaches as in [3] become not explicitly achievable. For this, other works propose optimisation program methods based on heuristic algorithms as in [4]. Other approaches integrate traffic simulation models in order to get the optimal coordinated timings for intersections as in [5].

The Multi-Agent System (MAS) has been provided on many applications from robotics engineering to economics and social sciences [6]. MAS consist of a set of dynamical autonomous agents interacting between each other to solve a complex task through collaboration. A representation of UTN systems using MAS is introduced in [7]. We can also find in the literature, approaches that apply Petri Nets (PNs) in UTN modelling, simulation or optimization [8]. The research works presented in [9] use timed and colored PNs to describe UTN, where the token represent the vehicle position. The initial attempts which have used Hybrid Petri Nets (HPNs) model can be found in [10]. The combination of Petri-nets and real-time multi-agent systems, for agent's communication modelling, is introduced in [11].

It's proved in [12] that the traffic light control system is one of the most profitable solutions for transportation problems, by improving the policies that control traffic lights. Urban traffic flows can be improved for a considerably lower cost, by minimizing the total delay at intersection [13]. We introduce several selected methods optimization-based signal traffic control for single intersection: Webster method [14], some heuristic or metaheuristic methods [15], differential evolution bacteria foraging algorithm [16], and Genetic algorithm [17]. For this context, this paper is devoted.

Moreover, some traffic researchers have done an effort to develop several simulation platforms to evaluate different traffic models and/or control UTN flows, most of them use graphic editors to draw, simulate and analyze the UTN capacity, maximize green wave band or minimize the delay, we can site as example: TSIS-CORSIM (Traffic Software Integrated System, CORridor SIMulation) [11] AIMSUN and SUMO (Simulation of Urban MOBility), MAXBAND, TRANSYT-7F, SYNCHRO, SCOOT, OPAC...etc.

The main objective of this paper is to give an efficient timing plan of TPNs that control a two phases intersection taken from UTN of medium-sized city; the TPNs are

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combined with agents of MAS in hybrid architecture. Each physical space of the intersection represents an agent, and the behaviour of the interconnected agents is supervised by the TPNs. Thus, the following points address the main contributions of this paper:

- Physical spaces are described with modular differential equations where a flow of vehicles can be defined. This model includes the effects of the traffic lights, the street and intersection saturation levels, the change of vehicle flows due to the quantity of cars and the congestion phenomena by the interconnections of the agents.
- The traffic light cycles are defined by TPN model, with the supervising possibility of these cycles.
- To estimate the delay of vehicles, a new timing plan, different to the current one, is optimized and proposed, using Continuous Genetic Algorithm, to represent transitions durations of the TPN model.
- The proposed plan is compared with the current one, fixed by the city's municipal authority, on SUMO software under different objective functions.
- The proposed hybrid model is compared with TSIS-CORSIM software's model on Matlab, in order to show the scalability of the approach.

The remainder of the paper is organized as follows: section II gives the general definition of TPNs. The problem statement is presented in section III, the hybrid model presentation for urban traffic flows control in a single intersection is presented in section IV. An example of real single intersection in medium-sized city is supplied in section V. Section VI presents the timing plan problem and its solution. The obtained experiments results are given in section VII. Finally, future work and conclusions are given in section VIII.

2. Timed PETRI NETS Definition

A Timed Petri-Net TPN; is a graph structure defined by the following six-tuple

$$TPN = (P, T, A, w, M_0, \delta) \quad (1)$$

$P = \{p_1, p_2, \dots, p_n\}$ is a finite set of n nodes in a graph, called Places.

$T = \{t_1, t_2, \dots, t_m\}$ is a finite set of m nodes called Transitions.

$A: A \subseteq (P \times T) \cup (T \times P)$ is a finite set of arcs; they connect transitions and places.

$w: A \rightarrow \mathbb{Z}^+$ is the positive integer weights of arcs.

M_0 is the initial marking or number of tokens contained in each place $p \in P$.

Let $M_m(p): P \rightarrow \mathbb{Z}^+$ defines the n -th vector with tokens number in each place at time tm . The tm -th PN marking is achieved according to the transition rule bellow. A transition $t_i \in T$ is said to be enabled in a PN with finite capacity if and only if:

$$M_m(p_j) \geq w(p_j, t_i), \forall p_j | w(p_j, t_i) \in w, \text{ such as } j = 1, \dots, n; \\ i = 1, \dots, m.$$

If t_i is enabled the next marking is defined as:

$$M_{m+1}(p_j) = M_m(p_j) - w(p_j, t_i) + w(t_i, p_j).$$

$\delta: T \rightarrow \mathbb{R}^+$ is the finite set of transitions time delays, each transition t_i in the net assigned by positive number τ_i such as: $\tau_i = \delta(t_i)$.

3. Problem Statement

Let $\beta = \beta_{st} \cup \beta_{cr} \cup \beta_{so} \cup \beta_{si}$ is the set of agents or physical spaces where the vehicles can be moved or placed in a UTN. $\beta_{st} = \{\beta_{st}^1, \dots, \beta_{st}^n\}$ is the set of n street agents, $\beta_{cr} = \{\beta_{cr}^1, \dots, \beta_{cr}^m\}$ is the set of m intersection agents, $\beta_{so} = \{\beta_{so}^1, \dots, \beta_{so}^k\}$ is the set of vehicle sources, i.e. buildings, parkings, garages...etc. $\beta_{si} = \{\beta_{si}^1, \dots, \beta_{si}^s\}$ is the set of vehicle sinks i.e. agents containing vehicles that leave the UTN. These allow us to construct all differential equations, explained below. A real example of a single intersection that illustrates all of these physical spaces is given in Figure 1, where the inputs and outputs of vehicles flow are respectively depicted by source and sink agents.

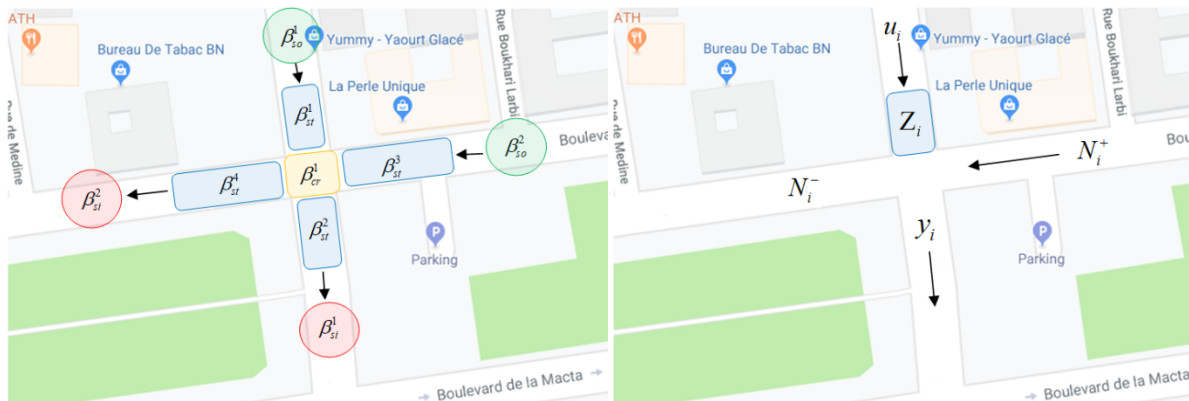


Fig. 1. Physical spaces in a real single intersection

Let $N_i^+ \subset \beta$ and $N_i^- \subset \beta$ are the in-adjacency and the out-adjacency sets of β_i , which represent all physical spaces providing and carrying out vehicles to and from $\beta_i \in \beta$ respectively. Streets, intersections, sources and sinks interconnections are assured using these two sets. $z_i \in \mathbb{R}$ is the number of vehicles in each agent $\beta_i \in \beta$, $\alpha(z_i) \in \mathbb{R}$, $\forall \beta_i \in \beta$ is the vehicles speed function that represents the inflow and outflow rates according to vehicles density change in each agent. Thus the hybrid model that will allows us to control vehicle density and distribution, and the optimal traffic lights, is consisting of:

- A modular and simple flow differential equations set for each z_i describing agent's occupation.
- A functions set $\alpha(z_i)$ to describe experimental data flows.
- A TPN set PN_i , $\forall \beta_{cr}^i \in \beta_{cr}$ for traffic light management.
- A supervisory control is guaranteed using the relationships set, between the TPN models and the differential equations is to feedback loop characterization.

4. Hybrid Model Presentation

This section gives a modular and mathematical representation of UTN, by developing a differential equations set that represent all the agents in the intersection. This will permit to get easier the complex UTN encompassing all intersection variants. According to section III, the modular model of vehicle flow for each agent, can be given by:

$$\dot{z}_i = \sum_{\beta_j \in N_i^+} u_{j,i} - \sum_{\beta_k \in N_i^-} y_{i,k} \quad (2)$$

$$y_{i,k} = a_{i,k}(z_k) \Gamma_{i,k} \alpha_i(z_i) z_i, \quad \forall \beta_k \in N_i^- \quad (3)$$

We put z_i as the vehicles quantity in $\beta_i \in \beta$, $u_{j,i}$ and $y_{i,k}$ are the input flow to agent β_i from $\beta_j \in N_i^+$ and the output flow from β_i to agent $\beta_k \in N_i^-$ respectively. Therefore, and according to the equation (2), the z_i dynamics are controlled by $u_{j,i}$ and $y_{i,k}$.

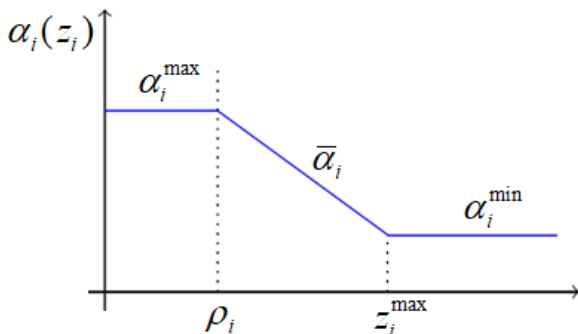


Fig. 2. The α_i change's with respect to z_i

From equation (3), $\alpha_i(z_i)$ is a function that expresses the vehicles unloading flow (Figure 2), it can be detailed as:

$$\alpha_i(z_i) = \begin{cases} \alpha_i^{\max}, & \text{if } z_i < \rho_i \\ \bar{\alpha}_i, & \text{if } \rho_i < z_i < z_i^{\max} \\ \alpha_i^{\min}, & \text{if } z_i = z_i^{\max} \end{cases} \quad (4)$$

From Figure 2, z_i^{\max} is the vehicles maximum density for each agent, we can define the linear decrease of α_i as:

$$\bar{\alpha}_i = \left(\frac{\alpha_i^{\min} - \alpha_i^{\max}}{z_i^{\max} - \rho_i} \right) (z_i - \rho_i) + \alpha_i^{\max} \quad (5)$$

Each agent is given by the physical length of streets, according to a minimum value $\alpha_i^{\min} < \alpha_i^{\max}$. The value of α_i^{\max} depends on the area of the agent β_i . If the number of vehicles in the agent increases, then the rate of unloading flow will decrease in this case. We can find many similar functions in the literature, as in [18]. Also, we define the activation functions $\Gamma_{i,k}$ and $a_{i,k}$ which activate and deactivate the output flows, $\Gamma_{i,k}$ is given by:

$$\Gamma_{i,k} = \begin{cases} c_{i,k} M_k, & \text{if } \beta_k \in \beta_{cr} \\ c_{i,k} M_i, & \text{if } \beta_i \in \beta_{cr} \\ 1, & \text{otherwise} \end{cases} \quad (6)$$

such as $M_k \in \mathbb{R}^{n_k \times 1}$ is the actual marking vector of the TPN with n_k places representing the traffic light behaviours for the single intersection β_k , and $c_{i,k} \in \mathbb{R}^{1 \times n_k}$ correspond to the traffic light cycle of each phase. Thus, the function $\Gamma_{i,k}$ is related to a traffic light cycle if the output $y_{i,k}$ connects the agent β_i to a single intersection β_k supervised by a traffic light, and it is obtained as a linear combination of the vector M_k . If there is no traffic light for β_k or $\beta_k \notin \beta_{cr}$, then $\Gamma_{i,k} = 1$ i.e. always activated, thus a free flow of vehicles.

The feedback information is represented by the functions $a_{i,k}(z_k)$ in order to control the output flow of β_i depending on the saturation level of the downstream agent $\beta_k \in N_i^-$, $a_{i,k}(z_k)$ is defined as:

$$a_{i,k}(z_k) = \frac{1}{1 + \exp\left(k_e \left(\frac{z_k}{z_k^{\max}} - \xi\right)\right)} \quad (7)$$

such as $k_e, \xi \in \mathbb{R}$. When β_k is full, $a_{i,k} \approx 0$ and the output flow of the precedent agent β_i is blocked. Finally, the equations (2)

and (3) are applied directly to streets and intersections i.e. $\forall \beta_i \in \beta_{st}, \beta_{cr}$, also to sources and sinks i.e. $\beta_i \in \beta_{so}, \beta_{st}$

The flows switching is assured by adding TPN to MAS-based model in the context of control theory, according to the scheme shown in Figure 3. Depending on certain traffic light cycle, the functions $\Gamma_{i,j}$ are activated by TPN and the feedback signals $a_{i,j}$ are transmitted from the MAS-based model. This communication ensures the supervisory control of the single intersection. The traffic light cycle has been defined by applying an optimization method to get the optimal timing plan. The timing optimization problem is presented in section VI. A real example of two phases intersection is presented immediately to illustrate the approach.

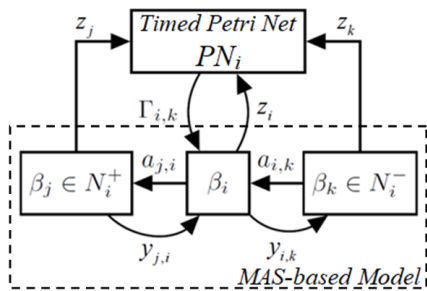


Fig. 3. The proposed Hybrid Approach

5. Case Study

The modular model of vehicle flow obtained in the previous section, is applied to a real intersection in Sidi Bel-Abbes, as an example of medium-sized city in Algeria, the intersection of Colonel Amirouch Ave and Mohammed Khmista Ave is composed by a simple four-road, with one bidirectional and one unidirectional, as shown in Figure 4. The studied intersection is fed by three vehicle sources and discharged by three sinks; it admits two phases signal control as illustrated in Figure 5. Using the formulation described in section IV, the intersection is redrawn as shown in Figure 6. Thus, $n = 6, m = 1, k = 3, s = 3$ so that, the subscripts are reordered: $\beta_{st} = \{\beta_1, \beta_2, \beta_3, \beta_4, \beta_5, \beta_6\}$, $\beta_{cr} = \{\beta_7\}$, $\beta_{so} = \{\beta_9, \beta_{11}, \beta_{12}\}$ and $\beta_{si} = \{\beta_8, \beta_{10}, \beta_{13}\}$. Since β_7 has traffic light, a corresponding TPN PN_7 is set. According to the agents' interconnection shown by the arrows in Figure 6, the input and output adjacency subsets for agent β_7 (as example) can be given by:

$$N_7^+ = \{\beta_1, \beta_3, \beta_5\}, N_7^- = \{\beta_2, \beta_4, \beta_6\} \tag{8}$$



Fig. 4. The studied real single intersection.

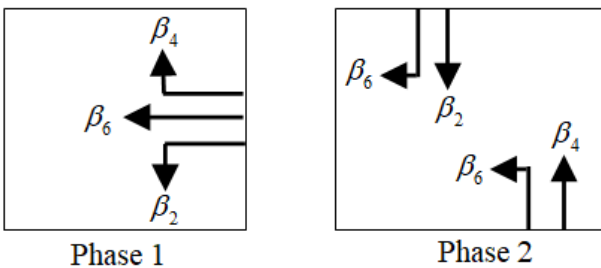


Fig. 5. The two phases of studied intersection.

- r = effective red time (sec)
- R = displayed red time (sec)
- S = departure (saturation) flow rate from queue during effective green (veh/sec)
- t = time
- Td = duration of analysis period in time dependent delay models
- x = degree of saturation, $x=(q/S) / (g/c)$, or $x = q/C$
- y = flow ratio, $y=q/S$
- Y = yellow (or clearance) time (sec)

In this work, we assume that “C” is not constant over time and is not priori unknown unlike in the work [17] where the authors consider the cycle length is constant over time and prior known, whether traffic demand. Where this hypothesis is restricted, however, lengths of particular phases (i.e., green times) can change over time; they are the decision variables in our optimal control. For this, delay is minimized according to the following equations of the U. S. Highway Capacity Manual (2000):

$$\sum_{j=1}^J d_{j,Td} \tag{11}$$

$$d = d_{un} * DF + d_{ov} \tag{12}$$

With:

$$d_{un} = \frac{C(1-g/C)^2}{2(1-xg/C)} \tag{13}$$

$$d_{ov} = 900 flx^2 \left[(x-1) + \sqrt{(x-1)^2 + \frac{mx}{cfl}} \right] \tag{14}$$

Where DF is the delay factor to account for signal coordination and controller type; m is the calibration parameter which depends on the arrival pattern and fl is the flow period in hours ($fl=0.25$ in 1994 HCM). Finally, the timing optimization problem for model (11) is formulated as following:

$$\text{Minimize}_{C, g_i, Td} \left(\sum_{j=1}^J d_{j,Td} \right) \tag{15}$$

Such as:

$$\sum_{i=1}^I g_{i,Td} = C \tag{16}$$

$$g_{i,Td} \geq g_{i,min}, \quad g_{i,Td} \leq g_{i,max} \tag{17}$$

$$C \geq C_{min}, \quad C \leq C_{max} \tag{18}$$

$$g_{i,max} < C \tag{19}$$

Where: $g_{i,min}$ is the minimum green time(s); $g_{i,max}$ is the maximum green time(s); C_{min} is the minimum cycle length time(s); C_{max} is the maximum cycle length time(s); in equation (12) the function is non-linear and non-convex, that makes it difficult to obtain the best and the global minimums. In our case these minimums are obtained using Continuous Genetic Algorithm (CGA) [19], it is an adaptation of Genetic Algorithm (GA) to the continuous optimization problems.

Especially, CGA uses a type of real coding, which is as close as possible to Hollands approach using binary coding. The algorithm starts with an initial population of “ n ” individuals: an individual is composed of real coordinates, respectively associated to the variables of the objective function at hand. The reproduction operators, inspired by the genetics, are applied to this population; offspring are created from parents. The new population is constituted in selecting the best individuals. By repeating this process, one hopes to enrich gradually the population with the most efficient individuals. The usual mechanisms of reproduction are the crossover, which consists in exchanging some coordinates of two individuals, and the mutation, which consists in generating a new coordinate at a given place of one individual.

CGA allows us to resolve the timing optimization problem, each individual in CGA stands for a timing scheme of the intersection, as, $g_i = \{g_1; g_2; \dots; g_i\}$ in which g_i represents real green time of phase i in the scheme and it is the variable remaining to be optimized according to general steps of the algorithm presented below.

Algorithm 1: General CGA

<< **Initialisation** >>;

Define cost function, and variables;

Select GA parameters;

Generate initial population;

<<Chromosome_Cost>>, Evaluation of the fitness of each chromosome in the population;

Select Mates for the reproduction;

Crossover and mutation;

<<Convergence_Check>>.

if the criteria is satisfied then

Return the solution;

else

Return to <<Chromosome_Cost>>;

end if

End

7. Experiments Results

The hybrid model is built on Matlab, and the data are simulated using SUMO and TSIS-CORSIM Software. The real data were collected from the real single intersection (section V), where the sources $\beta_9, \beta_{11}, \beta_{12} \in \beta_{so}$ provide approximately 700, 953 and 140(veh/h) respectively. The street agents $\beta_1, \beta_2, \beta_3, \beta_4, \beta_5, \beta_6 \in \beta_{st}$ and $\beta_7 \in \beta_{cr}$ are initially empty. The maximum occupancy of vehicles in each agent $\beta_i \in \beta_{st}$ is defined as: $z_1^{max} = 110$, $z_2^{max} = 46$, $z_3^{max} = 46$, $z_4^{max} = 110$, $z_5^{max} = 17$, $z_6^{max} = 20$ and $z_7^{max} = 22$, in the case of sources $z_9^{max} = 800$, $z_{11}^{max} = 1050$, $z_{12}^{max} = 240$. According to the Current Traffic Signal Control Plan (CTSCP) which is fixed by municipal authority of Sidi Bel Abbes city, time

delays of PN₇ (Figure 7) are given by $\delta(t_1)=35$, $\delta(t_4)=45$, $\delta(t_2)=\delta(t_5)=5$ and $\delta(t_3)=\delta(t_6)=5$, so that $g_1=45$ sec and $g_2=35$ sec (green light of phase1 and phase 2). CTSCP and our Proposed Traffic Signal Control Plan (PTSCP) are simulated using SUMO software. The obtained results are presented in the Table 1. The Table 2 shows the parameters of the vehicles such as waiting time, loss time, CO2 and CO emission rate, NOx nitrogen oxide and noise under PTSCP and CTSCP. According to Table 2 the results shows that that PTSCP surpass clearly the CTSCP for all objective functions.

Table 1. PTSCP and CTSCP of green Lights

Phases	Phases duration (sec)	
	PTSCP	CTSCP
Phase 1	20	35
Phase 2	40	45
Cycle length	60	80

Table 2. Numerical Comparison between PTSCP and CTSCP

Objective functions	PTSCP	CTSCP	Improvement (%)
Waiting time (sec)	31	54	42.59
Time loss (sec)	38	85	55.29
CO2 (mg/sec)	3986.31	6081.51	34.45
CO (mg/sec)	65	112	41.96
NOx (mg/sec)	1.79	2.72	34.19
Noise (db)	46.13	49.41	6.63

Figures 8 to 10 presents the simulation results of agents behaviours obtained from Matlab. Figure 8 describes the number of vehicles changing with respect to time in the street agents $\beta_1, \beta_3, \beta_5 \in \beta_{st}$ i.e. the input flow coming from $\beta_9, \beta_{11}, \beta_{12} \in \beta_{so}$ respectively. Figure 9 and 10, present the behaviour of intersection agent, $\beta_7 \in \beta_{cr}$ and the occupancy of $\beta_2, \beta_4, \beta_6 \in \beta_{st}$ respectively. According to the obtained results shown in these figures, we can observe that all agents β_i do not saturate due to our sequence of PTSCP (Table 1) of traffic lights.

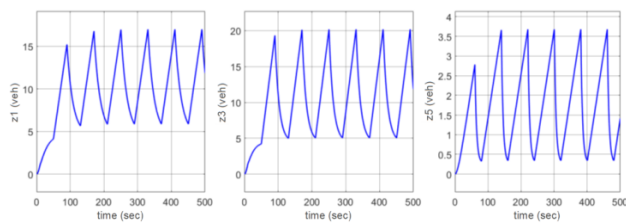


Fig. 8. The behaviour of agents $\beta_1, \beta_3, \beta_5 \in \beta_{st}$.

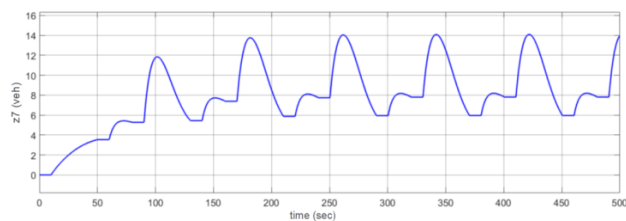


Fig. 9. The behaviour of agent $\beta_7 \in \beta_{cr}$.

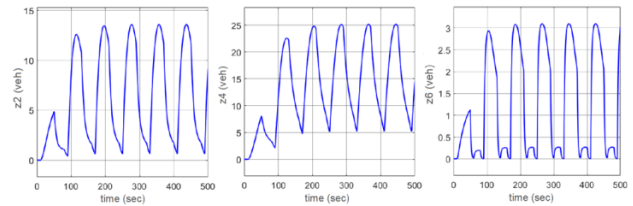


Fig. 10. The behaviour of agents $\beta_2, \beta_4, \beta_6 \in \beta_{st}$.

Another simulation of the studied example was carried out to formalize the hybrid approach through the simulation software package, TSIS-CORSIM. The figures below were obtained using the same real data. The input and output flows provided by street agents $\beta_1, \beta_3, \beta_5 \in \beta_{st}$ and $\beta_2, \beta_4, \beta_6 \in \beta_{st}$ of the hybrid-based model approach, are compared with TSIS-CORSIM software's flows and the results are illustrated by red graph and bleu one respectively on Figure 11 and Figure 12.

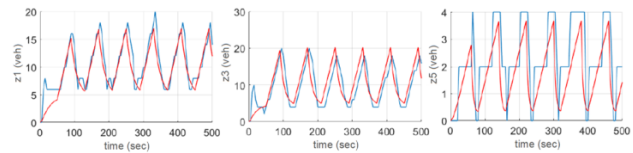


Fig. 11. The input flows from $\beta_1, \beta_3, \beta_5 \in \beta_{st}$.

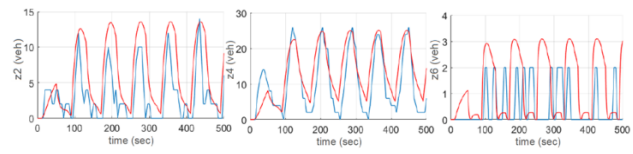


Fig. 12. The output flows to $\beta_2, \beta_4, \beta_6 \in \beta_{st}$.

8. Conclusions

We have presented in this paper an optimal supervisory control of a two phases intersection, taken from UTN of medium-sized city, through the TPN that control the single intersection. The TPN is combined with agents of MAS in hybrid architecture and it supervises effectively the behaviour of the interconnected agents due to the efficient timing plan (PTSCP) of the green lights. The green lights of each intersection phase optimized by CGA gives to the TPN an optimal performance to control traffic lights and, with consequence, intersection traffic flows, thus the network flows optimization.

This approach has been applied on a real single intersection chosen from the UTN of Sidi Bel-Abbes city because of its large capacity of sources, streets and sinks. The effectiveness and the scalability of the approach have been proven through the following two steps:

- The efficacy of the proposed timing plan (PTSCP) defined by CGA has been proven by comparing this plan with the current one (CTSCP) using SUMO software, under different objective functions.
- The results of the proposed hybrid model have been proved similar by comparing with the results of TSIS-CORSIM software's model.

Future work includes the development and simulation of a complex many-phases intersection in the same city network

and further experiments on SUMO and TSIS-CORSIM with real data to optimize the flows and improve the most important objective functions.

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