Jestr

Journal of Engineering Science and Technology Review 13 (6) (2020) 130 - 136

JOURNAL OF
Engineering Science and Technology Review

www.jestr.org

Research Article

Detailed Mathematical Analysis of Harmonics in Multi-Phase Inverters for Hybrid Electric Vehicles

Sangeeta Sahu*, Byamakesh Nayak and Rudra Narayan Dash

KIIT, Bhubaneswar, Odisha, 751024, India

Received 3 December 2019; Accepted 6 November 2020

Abstract

This paper presents a comparative analysis of power handling capability, harmonics, losses and cost of three phase, five phase and seven phase inverter which are the main criteria for selection of inverter applied to hybrid electric vehicle. The above performances of the multiphase inverter with different conduction angle are presented to trade off the best suited inverter for hybrid electric vehicle application. The 6^{th} harmonic pulsating torque created due to 5^{th} and 7^{th} harmonic component and 12^{th} harmonic pulsating torque created due to 11^{th} and 13^{th} harmonic which affect the smooth movement of the vehicle at the time of starting and braking condition. This paper gives a clear idea about the comparison of the above harmonic component for different conduction angle scenario of 3phase, 5phase and 7phase inverter. The equations of the performance criteria such output voltage, THD, power handling, losses, etc has been mathematically estimated and verified through Matlab Simulink Power Graphical User Interface.

Keywords: Harmonics, Multi-Phase Inverter, Phase Voltage, THD, Hybrid Electric Vehicle

1. Introduction

An inverter circuit topology uses two switches connected in series as one inverter arm. The number of inverter arms depends on number of phases. That is, a three phase inverter will have three inverter arms whereas a seven phase inverter circuit will have seven inverter arms. All the inverter circuits used in literature for multiphase inverters employ the same topology. In multi phase inverters, the conduction angle can be varied to get optimum output. Depending on the application, the switches used in inverters can be IGBT, MOSFET, GTO, etc. Each of the switches have diodes connected across them and these diodes act as feedback diodes which returns the energy stored in the inductive load to the dc supply. A comparative study on the total harmonic distortion of the output phase voltage at different conduction angle has been presented in this paper.

With the decrease in harmonic profile, the ripple in the torque produced in electric vehicles is reduced. For 5th and 7th harmonic, the torque ripple is pulsating with 6 times the fundamental frequency. For 11th and 13th harmonic, the torque ripple is pulsating at 12 times the fundamental frequency. Due to these harmonics, it produces noise, humming sound and jerking at the time of braking. For the same input voltage the power handling capability increases with the increase in the number of phases of the motor. But at the same time, the cost of inverter as well as of the control procedure increases with increase in number of phases. So, there must be a tradeoff between power handling capability, THD and cost of inverter. The 5^{th} and 7^{th} phase inverters can be used in hybrid electric vehicles. Multisource inverter can drive traction motors from variable DC voltages without using additional power converters has application in electrified power trains [1]. The transient analysis of high power VSI shows that a VSI is always a stable system with nonlinearities and unstable with respect to control signal [2]. The THD and switching losses are reduced considerably a dual voltage source inverter which is implemented through a modified space vector modulation [3]. A six phase current reconstruction scheme for dual inverters that have application in hybrid electric vehicle is presented in [4]. In this method the simulation and experimental results proof that the cost and volume of dual inverters are reduced as less number of current sensors are used. A bidirectional Z – source inverter has been recommended for the hybrid electric vehicle drive system in which the DC-link voltage is controlled such that the permanent magnet synchronous motor can operate at high speed without weakening the field [5]. The magnetic field created by the inverter in electric vehicles is evaluated using finite element method [6] and few design guidelines have been provided to minimize the same.

The fast development of power electronic devices and modern control theory had made the study and applications of five phase permanent magnet synchronous motor as well as its driving system that is the voltage source inverters attract more and more attention. Five phase PMSM has wide applications requiring high DC bus voltage utilization, reliability and power output [7]. THD comparison for a three phase inverter at different conduction mode concludes that 150 deg conduction mode has least THD [8-9]. PWM operation of a five phase voltage source inverter with hysteresis controller can be applied to a variable speed induction motor drive under asymmetrical connections [10]. A comparative study of performance of five phase three level inverter with five phase two level inverter for equal loading shows that five phase three level gives better performance [11]. A five phase inverter fed five phase star connected load operating with five different excitations has been simulated

*E-mail address: Sahu.sangeeta@gmail.com
ISSN: 1791-2377
doi:10.25103/jestr.136.18

and compared with that of three phase conventional inverter [12]. The ripple in torque and harmonics can be reduced by supply a motor with multi phase supply that is greater than three phase supply [13]. A multiphase inverter with phase-shifted control is applied in electric vehicles for inductive power transfer [14]. Here the inverter phase angle is adjusted to regulate the charging voltage. The implementation of space vector pulse width modulation for five phase inverter is very simplifies the computationally complexity [15-16]. Current imbalance between stator windings that results in shortening of the life of motor can be eliminated by symmetrical multiphase inverters [17].

2. Structure of a Multiphase Inverter

The multiphase inverter consists of multiple legs with two switches in each leg and the number of legs equals to the number of phases. Each switch has a diode connected in anti-parallel with it. The gating pulse in two adjacent switches has a delay of $(360/m)^0$ where n is the number of phases. The gate pulse between two switches of same leg has a delay of 180^0. The inverter is connected to a star connected resistive load. The schematic diagram of seven phase full bridge voltage source inverter is shown in Fig.1.

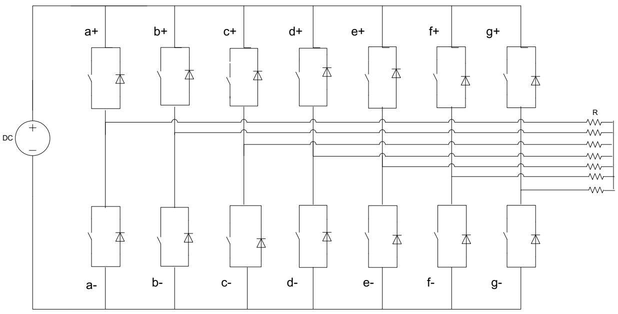

Fig. 1. Schematic diagram of Seven Phase Inverter

3. Conduction Techniques in Multiphase Inverters

In industries, variable frequency and voltages are required for various applications. The switching of the semiconductor switches in an inverter depends on the firing angle giving to the switches. Multiphase inverters are operated in different conduction modes. There are different modes on conduction in the multiphase inverters that decides the period for which the switches shall conduct. The conduction angle of a switch is given by the formula

$$\left(180 - \frac{360}{2m} P\right)$$

where P may vary from 0 to (m-1) and m is the number of phases.

For P = 0, that is 180 deg mode of conduction, each switch conducts for 180 deg. At any particular instant, all the switches are in ON state. Similarly for P = 1, (m-1) switches are in ON state at any instant. For P = ½, the switching sequence changes alternatively between n and (m-1) switches for each (360/4m) deg. Tables 1, 2 and 3 shows the switching sequence for P = 0, 1 and ½ for three phase, five phase and seven phase inverters respectively.

4. Output Voltage Waveform of Multiphase Inverters

The phase and line voltage waveform for 3, 5 and 7 phase inverters at P =0, ½ and 1 is shown in Figs. 2 and 3 respectively. At P = ½ conduction angle, the inverter can be considered as a seven level inverter as the phase voltage output has seven levels. The significant advantage of this output waveform is that it is close to sinusoidal. Since higher the number of levels, closer is the waveform to sinusoidal wave. This in turn reduces the stress on the motor's insulation level.

Table 1. Switching sequence for P = 0

Switching Mode	3 Phase	5 Phase	7 Phase	Switching Mode	3 Phase	5 Phase	7 Phase
1	$a^+ c^+$ b^-	$a^+ d^+ e^+$ $b^- c^-$	$a^+ e^+ f^+ g^+$ $b^- c^- d^-$	8	-	$c^+ d^+$ $a^- b^- e^-$	$b^+ c^+ d^+$ $a^- e^- f^- g^-$
2	a^+ $b^- c^-$	$a^+ e^+$ $b^- c^- d^-$	$a^+ f^+ g^+$ $b^- c^- d^- e^-$	9	-	$c^+ d^+ e^+$ $a^- b^-$	$b^+ c^+ d^+ e^+$ $a^- f^- g^-$
3	$a^+ b^+$ c^-	$a^+ b^+ e^+$ $c^- d^-$	$a^+ b^+ f^+ g^+$ $c^- d^- e^-$	10	-	$d^+ e^+$ $a^- b^- c^-$	$c^+ d^+ e^+$ $a^- b^- f^- g^-$
4	b^+ $a^- c^-$	$a^+ b^+$ $c^- d^- e^-$	$a^+ b^+ g^+$ $c^- d^- e^- f^-$	11	-	-	$c^+ d^+ e^+ f^+$ $a^- b^- g^-$
5	$b^+ c^+$ a^-	$a^+ b^+ c^+$ $d^- e^-$	$a^+ b^+ c^+ g^+$ $d^- e^- f^-$	12	-	-	$d^+ e^+ f^+$ $a^- b^- c^- g^-$
6	c^+ $a^- b^-$	$b^+ c^+$ $a^- d^- e^-$	$a^+ b^+ c^+$ $d^- e^- f^- g^-$	13	-	-	$d^+ e^+ f^+ g^+$ $a^- b^- c^-$
7	-	$b^+ c^+ d^+$ $a^- e^-$	$a^+ b^+ c^+ d^+$ $e^- f^- g^-$	14	-	-	$e^+ f^+ g^+$ $a^- b^- c^- d^-$

Table 2. Switching sequence for P = 1

Switching Mode	3 Phase	5 Phase	7 Phase	Switching Mode	3 Phase	5 Phase	7 Phase
1	a^+ b^-	$a^+ e^+$ $b^- c^-$	$a^+ f^+ g^+$ $b^- c^- d^-$	8	-	$c^+ d^+$ $a^- b^-$	$b^+ c^+ d^+$ $a^- f^- g^-$
2	a^+ c^-	$a^+ e^+$ $c^- d^-$	$a^+ f^+ g^+$ $c^- d^- e^-$	9	-	$d^+ e^+$ $a^- b^-$	$c^+ d^+ e^+$ $a^- f^- g^-$

3	b^+ c^-	$a^+ b^+$ $c^- d^-$	$a^+ b^+ g^+$ $c^- d^- e^-$	10	-	$d^+ e^+$ $b^- c^-$	$c^+ d^+ e^+$ $a^- b^- g^-$
4	b^+ a^-	$a^+ b^+$ $d^- e^-$	$a^+ b^+ g^+$ $d^- e^- f^-$	11	-	-	$d^+ e^+ f^+$ $a^- b^- g^-$
5	c^+ a^-	$b^+ c^+$ $d^- e^-$	$a^+ b^+ c^+$ $d^- e^- f^-$	12	-	-	$d^+ e^+ f^+$ $a^- b^- c^-$
6	c^+ b^-	$b^+ c^+$ $a^- e^-$	$a^+ b^+ c^+$ $e^- f^- g^-$	13	-	-	$e^+ f^+ g^+$ $a^- b^- c^-$
7	-	$c^+ d^+$ $a^- e^-$	$b^+ c^+ d^+$ $e^- f^- g^-$	14	-	-	$e^+ f^+ g^+$ $b^- c^- d^-$

Table 3. Switching sequence for P = ½

Switching Mode	3 Phase	5 Phase	7 Phase	Switching Mode	3 Phase	5 Phase	7 Phase
1	$a^+ c^+$ b^-	$a^+ d^+ e^+$ $b^- c^-$	$a^+ e^+ f^+ g^+$ $b^- c^- d^-$	15	-	$c^+ d^+$ $a^- b^- e^-$	$b^+ c^+ d^+$ $a^- e^- f^- g^-$
2	a^+ b^-	$a^+ e^+$ $b^- c^-$	$a^+ f^+ g^+$ $b^- c^- d^-$	16	-	$c^+ d^+$ $a^- b^-$	$b^+ c^+ d^+$ $a^- f^- g^-$
3	a^+ $b^- c^-$	$a^+ e^+$ $b^- c^- d^-$	$a^+ f^+ g^+$ $b^- c^- d^- e^-$	17	-	$c^+ d^+ e^+$ $a^- b^-$	$b^+ c^+ d^+ e^+$ $a^- f^- g^-$
4	a^+ c^-	$a^+ e^+$ $c^- d^-$	$a^+ f^+ g^+$ $c^- d^- e^-$	18	-	$d^+ e^+$ $a^- b^-$	$c^+ d^+ e^+$ $a^- f^- g^-$
5	$a^+ b^+$ c^-	$a^+ b^+ e^+$ $c^- d^-$	$a^+ b^+ f^+ g^+$ $c^- d^- e^-$	19	-	$d^+ e^+$ $a^- b^- c^-$	$c^+ d^+ e^+$ $a^- b^- f^- g^-$
6	b^+ c^-	$a^+ b^+$ $c^- d^-$	$a^+ b^+ g^+$ $c^- d^- e^-$	20	-	$d^+ e^+$ $b^- c^-$	$c^+ d^+ e^+$ $a^- b^- g^-$
7	b^+ $a^- c^-$	$a^+ b^+$ $c^- d^- e^-$	$a^+ b^+ g^+$ $c^- d^- e^- f^-$	21	-	-	$c^+ d^+ e^+ f^+$ $a^- b^- g^-$

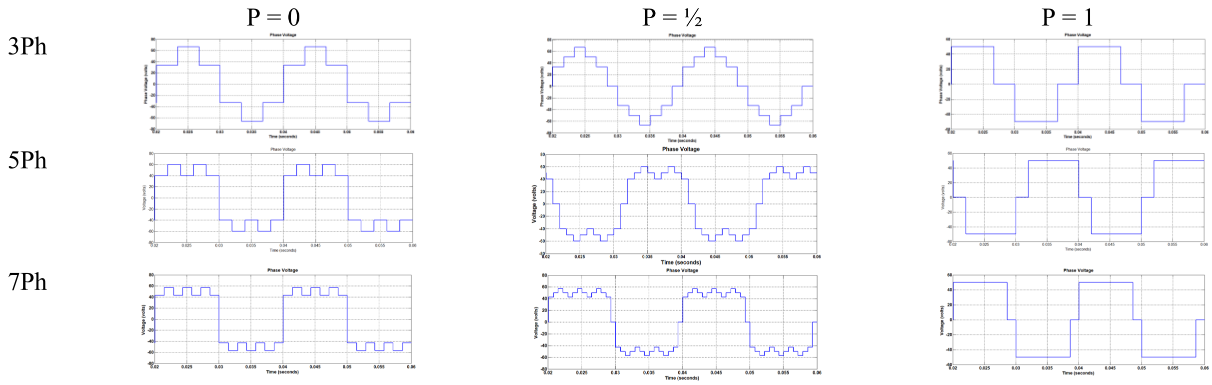

Fig. 2. Phase Voltage Waveform of Multiphase Inverters

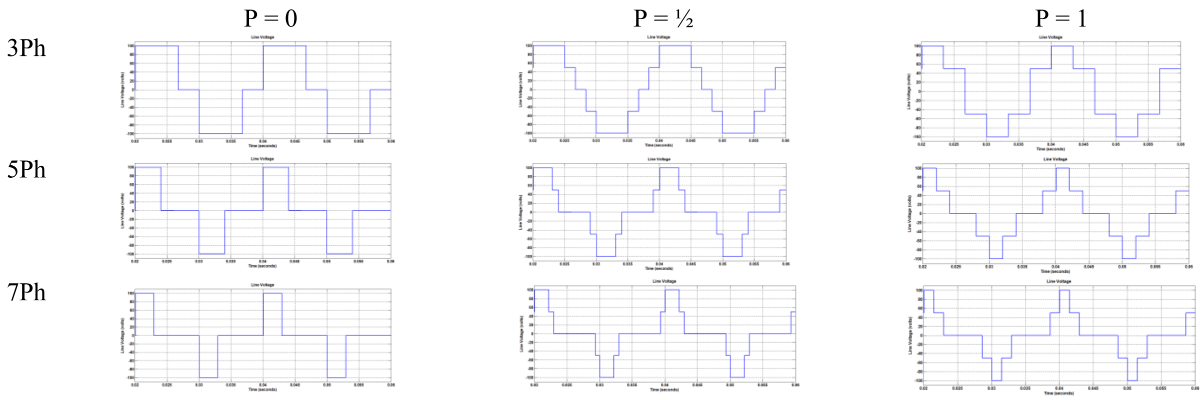

Fig. 3. Line Voltage Waveform of Multiphase Inverters

5. Results and Discussion

The phase voltage, line voltage, THD and loss comparison of three phase, five phase and seven phase inverters at P = 0, ½ and 1 are shown in Table IV. The table shows the simulation and the calculated values of various parameters of multiphase inverters. The phase voltage, line voltage and THD are calculated using the derived mathematical equations mentioned in the Appendix. The results are shown considering 1 p.u. input voltage. Here the switching loss, conduction loss and power handling capacity are considered as 1 p.u. for a three phase inverter. In a three phase inverter P = 0 conduction mode, three switches are in ON state at any instance. Thus conduction loss for three switches is considered as 1. Similarly at every 60 deg interval, one switch is changing its state. Thus switching loss for three phase inverter at P = 0 is considered as 1. Considering three phase inverter at P = 0 as reference, the switching loss, conduction loss and power handling capacity of five phase and seven phase inverters are calculated.

The power handling capability of the 3phase inverter is the lowest among higher phase inverter such as 5phase and 7phase. In order to handle the same amount of power, the stresses on the switches increases in 3phase inverter this reduces the life span of the inverter. In addition, the 5^{th}, 7^{th}, 11^{th}, 13^{th} harmonic components are more than the 5phase and 7phase inverter which creates 6^{th} and 12^{th} harmonic pulsating toque respectively that affect the machine performances. The 5 and 7 phase inverter may be considered for electric hybrid vehicle because of high power handling capability and less switching stress. The five phase inverter has no fifth harmonic and it's multiple whereas 7phase inverter has no seventh harmonic and it's multiple. The 7phase inverter has more 5^{th} harmonic component compared to 7^{th} harmonic component of 5phase inverter. Therefore it is concluded that the 6^{th} harmonic pulsating torque created by 5phase inverter is less compared to the 7phase inverter. Besides, the 11^{th} and 13^{th} harmonic of 5phase inverter is less as compared to the 7phase inverter which reduces the 12^{th} harmonic pulsating torque in 5phase inverter. The FFT analysis of 5phase inverter at conduction angle P = ½ (i.e. 162 deg) is shown in Fig. 4. The FFT analysis of 3, 5 and 7 phase inverters at different conduction angle P is obtained in a similar manner. Table V shows the harmonic profile of 3, 5 and 7 phase inverters and different conduction angle P. Figs. 5, 6 and 7 shows the graphical representation on multiphase inverters at P = 0, ½ and 1 respectively.

Table 4. Comparison of three phase, five phase and seven phase inverter at different conduction modes considering 1 p.u. input voltage

Parameter			**Phase Voltage in volts**	**Line Voltage in volts**	**THD in Phase Voltage in %**	**THD in Line Voltage in %**	**V_{0rms1}**	**V_{0rms}**	**Switching Loss**	**Conduction Loss**	**Power Handling Capacity**
Three Phase Inverter	P=0	Sim. Result	0.639	1.10	30.9	30.9	0.45	-	-	-	-
		Cal. Result	0.639	1.10	30.14	30.9	0.45	0.47	1	1	1
	P=1/2	Sim. Result	0.614	1.06	16.89	16.83	0.43	-	-	-	-
		Cal. Result	0.614	1.06	16.89	16.83	0.434	0.44	2	0.833	0.923
	P=1	Sim. Result	0.552	0.955	30.9	30.9	0.389	-	-	-	-
		Cal. Result	0.552	0.955	31.6	31	0.389	0.408	2	0.67	0.746
Five Phase Inverter	P=0	Sim. Result	0.639	0.748	42.93	65.45	0.45	-	-	-	-
		Cal. Result	0.637	0.728	43.14	65.23	0.45	0.449	1.67	1.67	1.67
	P=1/2	Sim. Result	0.628	0.738	33.6	53.02	0.442	-	-	-	-
		Cal. Result	0.615	0.748	32.9	52.99	0.442	0.464	3.33	1.5	1.54
	P=1	Sim. Result	0.605	0.711	30.19	42.93	0.43	-	-	-	-
		Cal. Result	0.605	0.692	29.87	42.912	0.43	0.428	3.33	1.33	1.49
Seven Phase Inverter	P=0	Sim. Result	0.638	0.559	45.54	92.11	0.451	-	-	-	-
		Cal. Result	0.638	0.559	46.36	92.10	0.45	0.496	2.33	2.33	2.33
	P=1/2	Sim. Result	0.632	0.552	38.42	80.55	0.447	-	-	-	-
		Cal. Result	0.632	0.552	39.1	80.57	0.447	0.48	4.67	1.859	2.28
	P=1	Sim. Result	0.621	0.541	33.76	68.8	0.439	-	-	-	-
		Cal. Result	0.621	0.541	33.51	68.8	0.439	0.463	4.67	2	2.2

Table 5. Harmonic Profile of Multiphase Inverter

		n = 3	**n =5**	**n =7**	**n =9**	**n =11**
P = 0	3Ph	0	12.62	9.18	0	5.69
	5Ph	21.2	0	9.09	7.07	5.79
	7Ph	21.22	12.66	0	7.13	5.77
P = 1	3Ph	0	10.85	8.03	0	4.84
	5Ph	12.46	0	5.34	6.73	5.51

	7Ph	16.75	5.78	0	2.81	4.34
P = ½	3Ph	0	3.39	2.27	0	5.61
	5Ph	18.89	0	4.13	1.11	0.91
	7Ph	20.02	10.77	0	3.89	2.02

```
Sampling time      = 5e-05 s
Samples per cycle = 400
DC component       = 1.259e-09
Fundamental        = 62.82 peak (44.42 rms)
THD                = 33.60%

  0 Hz  (DC):          0.00      90.0°
 25 Hz                 0.00       0.0°
 50 Hz  (Fnd):        62.82     152.6°
 75 Hz                 0.00       0.0°
100 Hz  (h2):          0.00       0.0°
125 Hz                 0.00       0.0°
150 Hz  (h3):         18.89      97.7°
175 Hz                 0.00       0.0°
200 Hz  (h4):          0.00       0.0°
225 Hz                 0.00       0.0°
250 Hz  (h5):          0.00       0.0°
275 Hz                 0.00       0.0°
300 Hz  (h6):          0.00       0.0°
325 Hz                 0.00       0.0°
350 Hz  (h7):          4.13     -12.2°
375 Hz                 0.00       0.0°
400 Hz  (h8):          0.00       0.0°
425 Hz                 0.00       0.0°
```

Fig. 4. FFT analysis of 5phase inverter at conduction angle P = ½ (162^0)

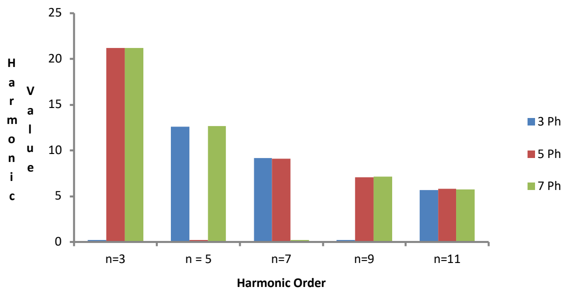

Fig. 5. Graphical representation of harmonic profile at P = 0

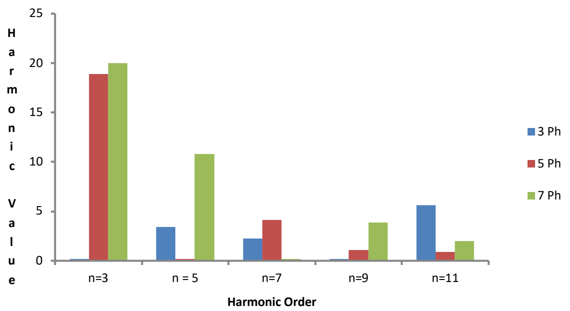

Fig. 6. Graphical representation of harmonic profile at P = ½

Although the power handling capability of 5phase inverter is less but the cost, switching losses and the 6^{th} and 12^{th} harmonic pulsating torque are also less compared to the 7phase inverter which may be the right choice for hybrid electric vehicle. In addition, the conduction period of each switch for P = 0, ½, and 1 are compared and found that the conduction period for P = ½ is the best choice for the above application. For P = ½ conduction angle, the level of phase voltage is more which can help to eliminate the number of dominant harmonic component using selective harmonic elimination technique. For example; if level is t, (t-1) harmonics can be eliminated. Fig. 8 shows the power handling capacity of three phase, five phase and seven phase inverters graphically at different conduction angle P.

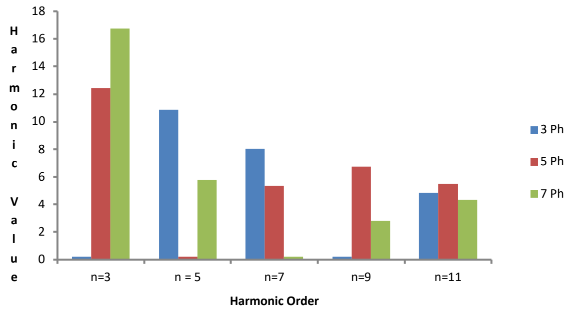

Fig. 7. Graphical representation of harmonic profile at P = 1

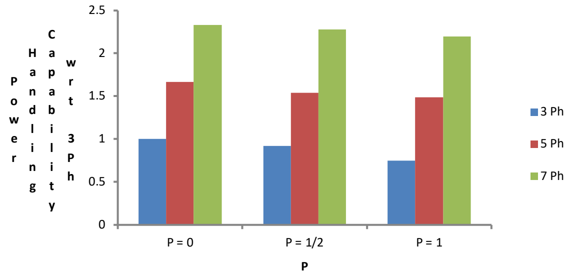

Fig. 8. Graphical representation of Power handling capacity of three phase, five phase and seven phase inverter at different P

6. Conclusion

The third harmonic voltage is absent when the neutral in a star connected load is isolated. Therefore, the harmonic analysis of multiphase inverters is performed without considering the third harmonic voltage. For P = ½, the 11^{th} and its higher order harmonics are less in a five phase inverter as compared to three phase and seven phase inverter. Also, it can be noted that all the nth order harmonics in the five phase inverter is less than the seven phase inverter. The power handling capacity of a five phase inverter is more than the three phase inverter but less than the seven phase inverter. The fifth order harmonics are absent in five phase inverter. The switching and conduction loss increases with the increase in the phase of the inverter due to higher number of switches used. It is worth noting that the calculated values of the output voltages, harmonics and THD approximately using the derived mathematical equations match with the results found from matlab simulink.

References

1. Lea Dorn-Gomba, Pierre Magne, Benjamin Danen and Ali Emadi, "On the Concept of the Multi-Source Inverter for Hybrid Electric Vehicle Powertrains," *IEEE Trans. on Power Electronics,* vol. 33, no. 9, pp. 7376 – 7386, 2018
2. Madhwi Kumari, Parashuram. R. Thakura and Deepak N. Badodkar, "Transient Analysis of Three-Phase High-Power Voltage Source Inverter With Nonlinearities in Hybrid Electric Vehicles," *IEEE Trans. on Power Electronics*, vol. 33, no. 4, pp. 3672 – 3680, 2018
3. Abbas Dehghani kiadehi, Khalil El Khamlichi Drissi and Christophe Pasquier, "Angular Modulation of Dual-Inverter Fed Open-End Motor for Electrical Vehicle Applications," *IEEE Trans. on Power Electronics*, vol. 31, no. 4, pp. 7376 – 7386, 2016
4. Haizhong Ye and Ali Emadi, "A Six-Phase Current Reconstruction Scheme for Dual Traction Inverters in Hybrid Electric Vehicles With a Single DC-Link Current Sensor," *IEEE Trans. On Vehicular Technology*, vol. 63, no. 7, pp. 3085 – 3093, 2014
5. P. Liu H.P. Liu, "Permanent-magnet synchronous motor drive system for electric vehicles using bidirectional Z-source inverter," *IET Electrical Systems in Transportation,* vol. 2, no. 4, pp. 178–185, 2012
6. Pablo Concha Moreno-Torres, Jerome Lourd, Marcos Lafoz and Jaime R. Arribas, "Evaluation of the Magnetic Field Generated by the Inverter of an Electric Vehicle," *IEEE Trans. on Magnetics,* vol. 49, no. 2, pp. 837 – 844, 2013
7. S. Wei et al., ''Overview on fault disposal and fault-tolerance mechanism of PMSM and its driving system,'' *Electr. Power Automat. Equip.*, vol. 36, no. 10, pp. 100–107, 2016.
8. Abhishek M. Patel, "THD Comparison for 180, 120 & 150 Degree Conduction Mode of Three Phase Inverter," in *International Journal for Scientific Research & Development*, vol. 6, pp. 145 – 149, 2018
9. Jaimin Trivedi, Manori Shah, Jui Shah and Ruchit Soni, "Three Phase 150 Degree Mode of Conduction Voltage Source Inverter using Arduino," *International Journal of Engineering Research & Technology*, vol. 5, pp. 272-275, 2016
10. Hamid A. Toliyat, "Analysis and Simulation of Multi-Phase Variable Speed Induction Motor Drives Under Asymmetrical Connections," in Proc. IEEE Applied Power Electronics Conf (APEC), 1996, vol. 2, pp. 586 – 592.
11. Bheemaiah Ch, Utkal Ranjan Muduli and Ranjan Kumar Behera, "Performance Comparison of Five-Phase Three-Level NPC to Five-Phase Two-Level Voltage Source Inverter," *IEEE International Conference on Power Electronics, Drives and Energy Systems*, 2019.
12. M.A Inayathullaah and Dr. R. Anita, "Simulation of Five Phase Voltage Source Inverter with Different Excitation for Star Connected Load," *International Journal of Engineering and Technology,* vol. 6, no. 3, 2014, pp. 1573 – 1580.
13. K. P. Prasad Rao, B. Krishna Veni and D. Ravithej, "Five-Leg Inverter for Five-Phase Supply," *International Journal of Engineering Trends and Technology,* vol. 3, pp. 144 - 152, 2012.
14. Qijun Deng, Ziyi Wang, Cheng Chen, Dariusz Czarkowski, Marian K. Kazimierczuk, Hong Zhou , and Wenshan Hu, "Modeling and Control of Inductive Power Transfer System Supplied by Multiphase Phase-Controlled Inverter," *IEEE Trans. on Power Electronics*, vol. 34, no. 9, pp. 9303 – 9315, 2019.
15. Chen Yong and Huang Qiu-Liang, "Four-Dimensional Space Vector PWM Strategy for Five-Phase Voltage Source Inverter," in *IEEE Access*, vol. 7, pp. 59013 – 59021, 2019.
16. Keng-Yuan Chen, "Space-vector pulse-width modulation for multiphase voltage source inverters considering reference order," *IET Power Electronics,* vol. 9, pp. 81 – 94, 2016.
17. Zicheng Liu, Zedong Zheng, Lie Xu, Kui Wang and Yongdong Li, "Current Balance Control for Symmetrical Multiphase Inverters," *IEEE Trans. on Power Electronics*, vol. 31, no. 6, pp. 4005 – 4012, 2016

Appendix

The mathematical equations for phase voltage and line voltage of three phase, five phase and seven phase inverters at P = 0, ½ and 1 are given below. The phase voltage is derived using Fourier transform.
RMS value of phase voltage at P = 0,

$$V_{phrms}=\frac{4V_{dc}}{3\sqrt{2}n\pi}\left[1+cos\frac{n\pi}{3}\right] \quad \text{3phase}$$

$$V_{phrms}=\frac{4V_{dc}}{5\sqrt{2}n\pi}\left[2+cos\frac{n\pi}{5}-cos\frac{2n\pi}{5}\right] \quad \text{5phase}$$

$$V_{phrms}=\frac{4V_{dc}}{7\sqrt{2}n\pi}\left[3+cos\frac{n\pi}{7}-cos\frac{2n\pi}{7}-cos\frac{4n\pi}{7}\right] \text{7phase}$$

RMS value of phase voltage at P = ½,

$$V_{phrms}=\frac{4V_{dc}}{3\sqrt{2}n\pi}\left[cos\frac{n\pi}{12}+\frac{1}{2}\left(cos\frac{3n\pi}{12}+cos\frac{5n\pi}{12}\right)\right] \text{3phase}$$

$$V_{phrms}=\frac{4V_{dc}}{5\sqrt{2}n\pi}\left[\begin{array}{l}2\,cos\frac{n\pi}{20}+\\ +\frac{1}{2}\left(cos\frac{3n\pi}{20}-cos\frac{9n\pi}{20}+cos\frac{5n\pi}{20}-cos\frac{7n\pi}{20}\right)\end{array}\right]$$

$$V_{phrms}=\frac{4V_{dc}}{7\sqrt{2}n\pi}\left[3cos\frac{n\pi}{28}+\frac{1}{2}\left(cos\frac{3n\pi}{28}-cos\frac{9n\pi}{28}-cos\frac{17n\pi}{28}+cos\frac{5n\pi}{28}-cos\frac{7n\pi}{28}-cos\frac{15n\pi}{28}\right)\right]\text{7phase}$$

RMS value of phase voltage at P = 1,

$$V_{phrms}=\frac{\sqrt{2}V_{dc}}{n\pi}cos\frac{n\pi}{6} \quad \text{3phase}$$

$$V_{phrms}=\frac{\sqrt{2}V_{dc}}{n\pi}cos\frac{n\pi}{10} \quad \text{5phase}$$

$$V_{phrms}=\frac{\sqrt{2}V_{dc}}{n\pi}cos\frac{n\pi}{14} \quad \text{7phase}$$

The RMS of fundamental value of phase voltage,

$$V_{phrms1}=\frac{V_{m1}}{\sqrt{2}}$$

The RMS value of line voltage,

$V_{L1} = 2 \times V_{phrms1} \times cos60$ 3phase

$V_{L1} = 2 \times V_{phrms1} \times cos54$ 5phase

$V_{L1} = 2 \times V_{phrms1} \times cos64.29$ 7phase

Power is given by,

P = $\frac{3V_{ph}^{\ 2}}{R}$ 3phase

P = $\frac{5V_{ph}^{\ 2}}{R}$ 5phase

P = $\frac{7V_{ph}^{\ 2}}{R}$ 7phase

The general equation of n phase inverter at P = 0 and 1 can be concluded from the above equations.

At P = 0,

$$V_{phrms} = \frac{4V_{dc}}{\sqrt{2}mn\pi}\left[\frac{m-1}{2} + cos\frac{n\pi}{m} - \sum_{l=2,4,6...\frac{m-3}{2}} cos\frac{ln\pi}{m}\right]$$

AAt P = 1

$$V_{phrms} = \frac{2V_{dc}}{n\pi}cos\frac{n\pi}{2m}$$

Here, m = number of phases and n = harmonic order