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Research Article

Sparse Vectors Based Load Flow Solution in Radial Power Distribution Systems

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Abstract

A load flow technique for the radial power distribution (RPD) system is proposed to find the receiving bus voltages using distribution system lines impedance and the sum of the load currents flowing throw the distribution line. The RPD system structure identifies by forming the three sparse matrices (vertical vectors without any zeros) to determine the buses connected to a particular distribution system line. The method is implemented on the 33-bus and 69-bus RPD systems with realistic loads and compared the efficacy of the proposed method with available existing methods.

Keywords: Radial Power Distribution System, Load Flow Technique, Receiving End Voltage, Line Impendence, Load Currents.

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1. Introduction

The power distribution system (PDS) performance analysis is significant in delivering the power to consumers from the bulk power system. Most of PDS have used the single main feeder in practice and are designated as RPD system. RPD system is simple in design and low in cost for construction; due to this, the RPD systems are more popular.

Load flow solution (LFS) is one of the essential tools for power system planning, operation, and analysis and obtained the steady-state behavior of PDS such as consumer voltage profiles and phase angle of consumer voltages, power flow in distribution lines and power losses in the distribution lines. The following are the requirements for load flow calculation method:

- Low storage and high-speed, particularly for large systems for real-time and interactive applications.
- Reliability should high, particularly for ill-conditioned problems, outage studies, and real-time application
- Should be versatility and simplicity

Transmission system LFS is not able to apply to PDS due to a high R/X ratio and radial structure. The following load flow methods are successfully computed and discussed in the literature by classified broadly into three types:

- Newton Raphson (NR) and Fast Decoupled (FD) Method [1-2]
- Backward/Forward Sweep (BFS) Method [3-14]
- Direct Method [15-18]

Venkatesh et al. [1] have presented an LFS technique for PDS by developing 2nd order equations set to describe the RPD system with a large R/X ratio. The equation set contains 3*(nd-1) equations and variables for nd node PDS which formulated the Jacobian matrix. Equation sets of this technique solved using 1st order NR technique to compute the voltages of PDS under consideration. Tortelli et al. [2] have explained LFS with a modified FD method for meshed and radial PDS by using the complex per unit normalization, which will improve the performance of the conventional FD load flow method. In Ghosh and Das [3], and efficient load flow method is presented with simple recursive algebraic equations which are used in the BFS LFS for PDS by forming the two-dimensional matrix to identify the nodes beyond any branch. Loop analysis-based load flow technique presented [4] a continuation LFS by extending the existing method BFS based load flow method. In Alinjak et al. [5] studied the BFS based load flow technique by forming the incidence matrix using improving the breadth-first search method. ALHajiri et al. [6] have developed the load flow method to make the LFS fast and flexible for PDS. This method is developed a radial configuration matrix (RCM) and has used in the BFS based load flow technique.

Shakarami et al. [7] have introduced a recursive equation-based LFS for PDS. To identify the PDS structure, the authors used graph theory and developed the four constant matrices. The LFS has directly solved these four matrices and shows better solvability for the steady-state analysis of PDS with good convergence. Ghatak and Mukherjee [8] have discussed the LFS of PDS by the formation of the load impedance matrix (LIM) using the set theory and calculate the node voltages in a single step instead of adopting the procedure in conventional BFS based LFS. Satyanarayana et al. [9] have introduced an efficient LFS with state-space recursive algebraic equations that are used in the BFS LFS for PDS by considering the different voltage-dependent static load models. In Nagaraju et al. [10], a novel LFS is presented with trigonometric recursive algebraic equations with two single vectors formed using the sparse technique to identify the EDS nodes and branches which are used in the BFS LFS for PDS by considering the different voltage-dependent, realistic load models. The RPD load flow technique was performed on a single load current to bus voltage (LCBV) matrix using the backward/forward sweep method [11]. The LCBV matrix had

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more zeros as network complexity increased. Sharma et.al [12] proposed a graph theory to derive topological-order matrix which was used in load flow solution for RPS. There is a need to follow steps in series to form the topological matrix. Ouali et al. [13] presented a derivation matrix that organized the derivation branches originating from the main branch's common buses. The same matrix is used to perform the load flow solution for RPD using the backward/forward sweep method. The derivation matrix is not a sparse matrix. Kawambwa et al. [14] used linear equations based on Kirchhoff's laws without involving matrix multiplication to perform the load flow solution using the conventional backward/forward sweep technique. There is a need to check parent and child nodes while performing load flow solutions.

The direct load flow techniques [15 – 17] have to defeat the drawbacks related to the traditional LFM and conventional BFS LFM. The distribution load flow (DLF = BCBV×BIBC) has obtained based on the branch current to bus voltage (BCBV) matrix and bus injection to branch current (BIBC) matrix to achieve better results of LFS for PDS. For radial systems, a fast and simple distribution load flow algorithm was proposed to compute BIBC matrix without the use of any additional algorithms [18].

A load flow technique for RPD system is proposed for identifying the branches connected to a particular branch using sparse technique and forming the three single vectors. Receiving consumer node voltages are computing by a simple iterative process and also finding branch losses and total EDS power losses. LFS is accelerating by considering the initial voltages and power losses in the initial iteration itself. The proposed method executed on 33 node and 69 node RPD systems with realistic loads and compared with the existing method in literature which gives the better improvement in reducing the memory store the RPD structure details and also acceptable number of iterations in normal conditions, ill-conditions and heavy loading conditions.

2. Formation of Sparse Vectors for PDS

The fig. 1 is the 3-phase balanced RPD system sample single line diagram, which shows the buses in bold numbers and lines are showing in normal numbers. The nd and ln are representing the number of buses and the number of lines in the RPD system, respectively.

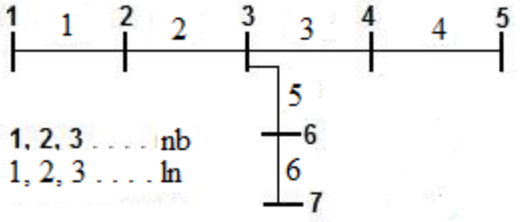

Fig. 1. RPD system single line diagram

Path line bus vector plb[ ] has formed to store the bus numbers which are connected to the all distribution system line. Based on the PDS structure vector, plb[ ] size will be changed. Two equal single dimension vectors are formed such as index-plb-from, ipf[ ] and index-plb-to, ipt[ ] which works as points to vector plb[ ]. Three single dimension vectors successively control each path bus memory location-allocation, where ipf[ ] and ipt[ ] contain the data of starting memory allocation and path branch *st* memory location in the vector plb[ ]. Fig. 1 is shown sample PDS for identifying the buses in the PDS structure, and there is no dependency on the bus numbering with a sub-station bus number. Table 1 and Table 2 shows data stored for PDS in plb[ ], ipf[ ] and ipt[ ] vectors. Fig. 2 shows the flow chart for the formation of plb[ ], ipf[ ] and ipt[ ] vectors in PDS.

Table 1. plb[] vector formation

Index (n)	plb[n]	Line No (st)
1	**2**	1
2	**3**	
3	**4**	
4	**6**	
5	**5**	
6	**7**	
7	**3**	2
8	**4**	
9	**6**	
10	**5**	
11	**7**	
12	**4**	3
13	**5**	
14	**5**	4
15	**6**	5
16	**7**	
17	**7**	6

Table 2. ipf[ ] and ipt[ ] vector formation

Line no (st)	ipf[st]	ipt[st]
1	1	6
2	7	11
3	12	13
4	14	14
5	15	16
6	17	17

3. Load Flow Solution

In PDS, a typical line *st* is shown in fig. 3, which consists *s* as sending end bus and *t* as receiving end bus with voltages as $V_s\angle\delta_s$ and $V_t\angle\delta_t$ respectively along with line *st* impedance $Z_{st} = R_{st}+jX_{st}$. The available demand at bus *t* is $P_{Lt}+jQ_{Lt}$, the current and power flow in the distribution line *st* in PDS are I_{st} and P_t+jQ_t.

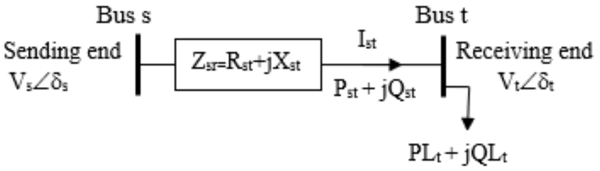

Fig. 3. Typical distribution line *st* in PDS

The load current at bus t is calculating with the below eqn. (1)

$$IL_t = \left(\frac{PL_t+jQL_t}{V_t}\right)^* \text{ for t} = 2, 3, 4, \dots \text{nb} \qquad (1)$$

The line current in line *st* is obtained by summing up all the load currents flowing through the line *st,* which can be calculated with eqn. (2)

$$I_{st} = \sum_{k=ipf(st)}^{k=ipt(st)} IL_{plb(k)} \text{ for st} = 1, 2, 3, \dots \text{Ln} \qquad (2)$$

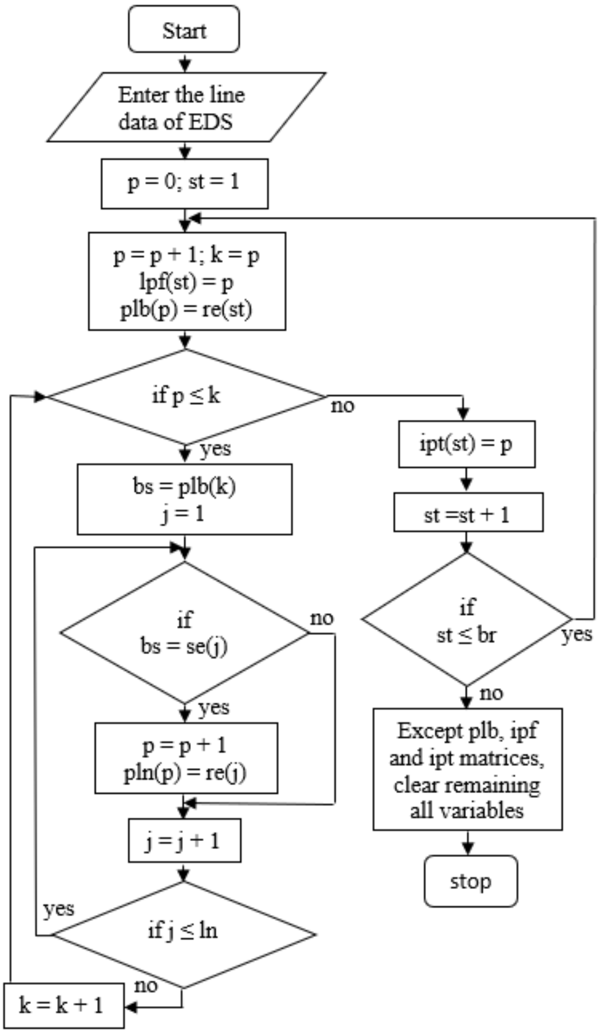

Fig. 2. Flow chart to form plb, ipf and ipt vectors

The voltage at bus *t* can be calculated using the below eqn. (3)

$$V_t = V_1 - \sum_{st=1}^{ln} \sum_{\substack{k=ipf(st) \\ if\ plb(k)=t}}^{k=ipt(st)} I_{st} * Z_{st} \text{ for t = 2, 3, 4, …. nb} \quad (3)$$

The active and reactive power losses can be calculated using the following eqns. (4) and (5)

$$LP_{st} = real(I_{st} Z_{st} I_{st}^{*}) \text{ for st = 1, 2, 3, …. ln} \quad (4)$$

$$LQ_{st} = img(I_{st} Z_{st} I_{st}^{*}) \text{ for st = 1, 2, 3, …. Ln} \quad (5)$$

The load flow solution of PDS is computed iteratively with simple eqns. (1) to (5) by considering the sparse vectors plb, ipf, and ipt. In the proposed method, consider all consumer buses initial voltages are 1 p.u. (same as substation voltage). The fig. 4 showing the complete details of the flow chart for the proposed method load flow solution for the RPD system.

5. Realistic Load Modelling

Active and reactive demands are constant in convectional PDS load flow study irrespective of amplitude change in voltage on the same bus. There are different types and categories of load models in the power system real-time operation. The nature of such active and reactive demands dependent on the frequency and voltage of the PDS and explore more in various planning scenarios. In paper considered for the load flow study using the constant power, industrial, residential and commercial loads as realistic loads [19]. Mathematically the load models are presented as follows

$$PL_t = PL_t^n * \left(\frac{|V_t|}{|V^n|}\right)^a \quad (6)$$

$$QL_t = QL_t^n * \left(\frac{|V_t|}{|V^n|}\right)^a \quad (7)$$

Where,
a and *b* are the load type exponent
PL_t^n and QL_t^n are the nominal voltage active and reactive power demands respectively at the receiving bus $|V^n|$ and $|V_t|$ are the nominal and receiving bus at *t* voltages respectively

Table 3 shows the active and reactive exponents for realistic loads. Evaluation of co-efficient PL and QL require to use of parameter estimation techniques

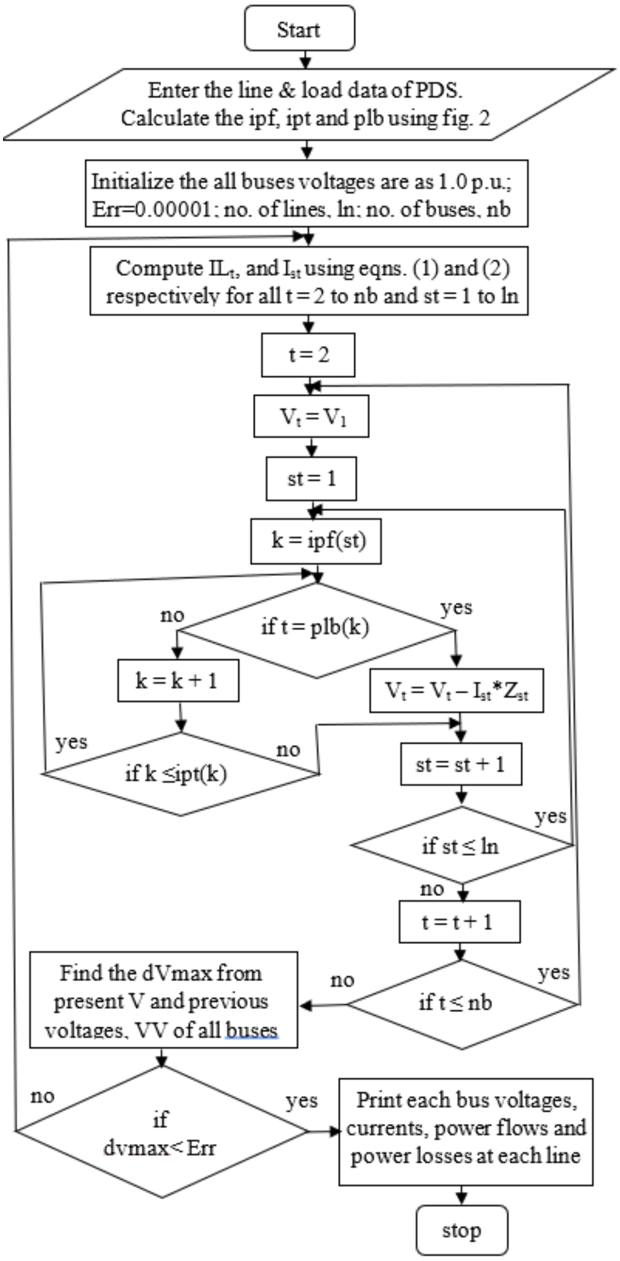

Fig. 4. RPD system Flow chart for Load flow calculation

Table 3. Various types of realistic loads in PDS

Realistic Load Type	*a*	*b*
Constant Power	0.00	0.00
Industrial	0.18	6.00
Residential	0.92	4.04
Commercial	1.51	3.40

While computing the industrial load effect, the PDS is assumed to be supplying only particular industrial consumers. Similarly, for residential and commercial loads, it is considered that all the loads are residential and commercial load model types, respectively. In real-time, we have different buses with any one of the industrial or residential or commercial load model, and details are discussed further while investigating the test cases.

5. Results and Analysis

The 33-bus and 69-bus RPD systems are considered [20] to show the effectiveness of the proposed method with the substation voltage of 12.66 kV. The recommended load flow is executed with realistic load models i.e., constant power, industrial, residential, commercial loads. Along with various individual realistic load models load flow solutions, it has also executed for the composite loads (with industrial, residential, and commercial loads) where the consumers have the different load types in 33-bus and 69-bus RPD systems. Table 4 is showing the details of various consumer buses considered for the various realistic load model type for 33-bus and 69-bus RPD systems.

Table 4. Consumer bus load model type used for composite loading in 33-bus and 69-bus RPD systems

RPDS	Consumer Bus Number		
	Industrial Load	Residential Load	Commercial Load
33-bus	2 to 4, 14, 18 to 23, 29, 31	5, 6, 9 to 13, 15 to 17, 26 to 28, 33	7, 8, 24, 25, 30, 32
69-bus	7 to 10, 16 to 18, 24, 28, 29, 36, 37, 39, 40, 45, 46, 48, 51, 54, 55, 62, 65, 68, 69	6, 13, 14, 20, 22, 26, 27, 33 to 35, 41, 43, 52, 53, 66, 67	11, 12, 21, 49, 50, 59, 61, 64,

The load flow results are presented in the table 5 and shown the total active power load (TAP_L) in kW and total reactive power load (TRQ_L) in kVAr available in the system, minimum voltage (Vmin) in p.u. along with bus number, total active power losses (TAP_{Loss}) in kW and total reactive power losses (TPQ_{Loss}) in kVAR in the system, number of iterations and execution time (ET) in sec are shown for each realistic load model.

From table 5, it has observed that commercial loads have less load impact, higher minimum voltage, and lower the losses compared with other realistic loads. The higher loads, lower minimum voltage and more losses in constant power loads. Industrial load, residential load, and composite loads are in between constant power load and commercials loads. There is no change in the consumer bus number for the minimum voltage, and load modeling is not impacting the minimum voltage consumer bus number. Load flow solutions for Industrial load and composite load models are taking a higher number of iterations for convergence, and the industrial load model is taking high execution time for load flow solution. Fig. 5 and fig. 6 are showing the voltages profile at consumer buses in 33-bus and 69-bus RPD systems, respectively.

Table 5. Summary of test results for 33-bus and 69-bus RPD systems

RPD system	Parameter	Constant Power Load	Industrial Load	Residential Load	Commercial Load	Composite Load
33-bus	TAP_L in kW	3715.00	3683.58	3558.60	3466.29	3549.82
	TRP_L in kVAr	2300.00	1704.70	1875.33	1939.83	1876.92
	V_{min} in p.u.	0.90378	0.91526	0.91601	0.91755	0.91595
	(Bus at V_{min})	(18)	(18)	(18)	(18)	(18)
	TAP_{Loss} in kW	210.97	167.79	164.54	159.49	164.78
	TRP_{Loss} in kVAr	143.12	113.62	111.24	107.70	111.45
	No. of iteration	3	4	3	3	4
	ET (sec)	0.058	0.069	0.066	0.065	0.068
69-bus	TAP_L in kW	3801.49	3770.94	3651.92	3565.87	3595.36
	TRP_L in kVAr	2694.60	2100.19	2274.37	2340.47	2306.15
	V_{min} in p.u. (Bus at V_{min})	0.90919 (65)	0.91875 (65)	0.92032 (65)	0.92221 (65)	0.92195 (65)
	TAP_{Loss} in kW	224.98	175.09	170.83	165.04	165.75
	TRP_{Loss} in kVAr	102.19	80.70	78.96	76.44	76.75
	No. of iteration	3	4	3	3	4
	ET (sec)	0.129	0.138	0.133	0.131	0.135

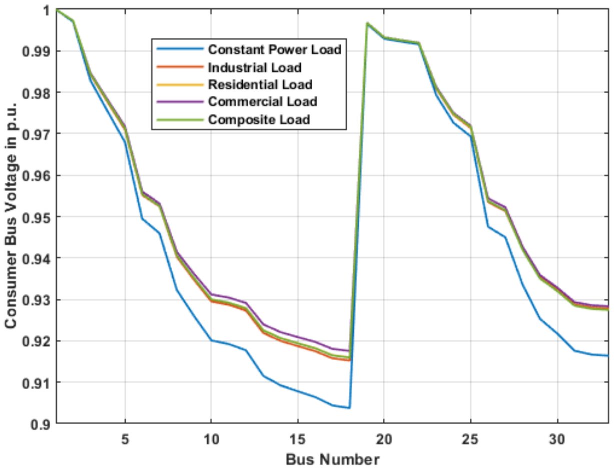

Fig. 5. Voltage profile at consumer buses in 33-bus RPD system

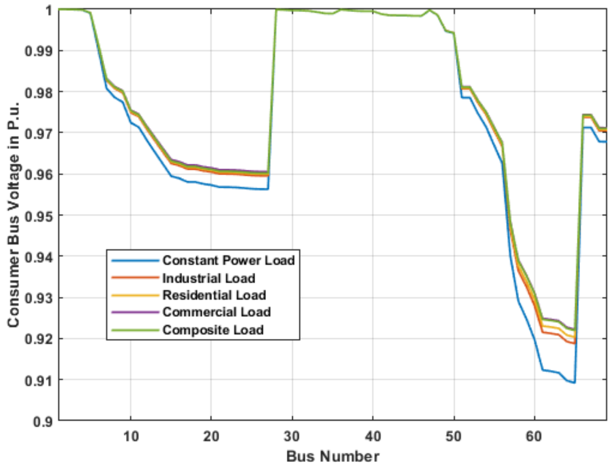

Fig. 6. Voltage profile at consumer buses in 69-bus RPD system

Table 6 shows the comparison of the proposed load flow method with the existing techniques in terms of execution time and the number of iterations for the constant power load model. The proposed method is taking the same number of iterations with a very slight time difference in existing methods.

Table 6. Comparison of the proposed method with existing methods in terms of execution time and number of iterations for constant power load

Name of the Load Flow Method	**33-bus RPD system**		**69-bus RPD system**	
	Execution time (sec)	**No. of Iterations**	**Execution time (sec)**	**No. of Iterations**
Proposed Method	0.058	3	0.129	3
Nararaju et.al. [10]	0.06	3	0.13	3
Satyanarayana et. al. [9]	0.06	3	0.13	3
Ghosh and Das [3]	0.09	3	0.16	3

6. Conclusions

In this paper, spare vectors have formed to reduce the space of memory to handle large RPD system structures, and a new load flow method has presented for the RPD system with simple algebraic equations solving iteratively. The proposed method has been discussed with 33-bus and 69-bus RPD systems with realistic load models and observed that high minimum voltage and lower RPD system power losses have shown in the commercial load model whereas low minimum voltage and high RPDS power losses have demonstrated in the constant power load model. Industrial, residential and composite load loads are in between the constant power and commercial load models with respect to minimum voltage and power losses. The effectiveness of the proposed method is compared in terms of execution time and the number of iterations; proposed method has slightly taken less time compared with existing methods.

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