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An Overview of Neutrosophic Topological Spaces and its Applications

V. Shyamaladevi and G. K. Revathi*

Department of Mathematics, School of Advanced Sciences, Vellore Institute of Technology Chennai, Chennai – 127, India

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Abstract

The goal of this paper is to examine various neutrosophic topological spaces and to learn about the differences between them. This paper provides the list of application of Neutrosophic topology in diverse fields like computer science, operational research, artificial intelligence and many sciences field. The importance of neutrosophic topological space in real time applications are listed in this article.

Keywords: Neutrosophic Set (NS), Neutrosophic Topological Spaces (NTS), Neutrosophic Soft Set (NSS), m-Polar Neutrosophic Topology (mPNT), Neutrosophic Soft Topological Spaces (NSTS), Pythagorean m-polar Fuzzy Neutrosophic Topology (PmPFNT).

1. Introduction

A common meaning of fuzzy is "not clear, distinct, or precise; blurred depending on their context." Fuzzy logic (FL) is a simple method for mapping an input space to an output space. The beginning of anything is a mapping of input to output. In 1965, Zadeh introduced the fuzzy set [1]. Atanassov defines FL as a replacement [2]. The real unit interval [0, 1] replaces the two-point set of standard value ranges 0, 1. Every real number in [0, 1] is thought to reflect a separate degree of accuracy that goes from 0, which is the invalid response in standard reason to 1, which is the correct answer. Conventional logic conjunctions appear as functions on the interval [0, 1]. For example $\mu_A \wedge \vartheta_A = \min(\mu_A, \vartheta_A)$, $\mu_A \vee \vartheta_A = \max(\mu_A, \vartheta_A)$ and are therefore assigned a phrase p with degree truth $\mu_A \in [0, 1]$, in FL is implicitly assumed to also have a degree of falsehood given by $(1 - \mu_A)$.

In general, intuitionistic fuzzy logic (IFL) does not need this to be true. IFS have been a significant research topic in the field of fuzzy sets (FS). The notion was originally introduced publicly around 1983, with the initial publically available citation occurring in 1986. According to Atanassov, an intuitionistic fuzzy set (IFS) is defined as an array of functions signalling membership (μ_A, ϑ_A) designated via IF, here ϑ_A represents the degree of non-belongingness in IF and μ_A represents the degree of belongingness [3].

It's also important to note that this concept is a fuzzification of a concept of sub-definite set, which was presented a few years ago from Narinyani, whose handles the set μ_A that contains elements recognised to belong to the sub-definite set separately, with the condition $\mu_A \wedge \vartheta_A = \varphi$. This has been expanded to the two membership functions μ_A and ϑ_A, which are anticipated to test the $\mu_A(x) + \vartheta_A(x) \leq 1$ condition for IFS.

Takeuti and Titani independently defined "IFS theory" as a set theory developed in intuitionistic logic [4]. Takeuti-Titani's IFL is just a modification of intuitionistic logic, hence any equations that are proven in intuitionistic logic are likewise verifiable in logic. The word "intuitionistic" was possibly inspired from the difference in Atanassov's notion of IFSs. As in intuitionistic logic, this is intended to express rejection of the excluded middle law. Such mathematical concepts look rational and intriguing from the standpoint on FS theory and also have implementation, however it may be argued that calling Atanassov theory IFSs is unsuitable and incorrect.

The primary objection of Atanassov's approach lies in the fact he refers to "IFS theory" as the acceptance of principles and rules that, once included in IL, render it typical, i.e. not anything about intuitionism exists. When Atanassov's theory is referred to as intuitionistic, an error occurs.

This is an excellent opportunity to introduce Smarandache's unique concept of the NS of truth values. The neutrosophic theory is a philosophical discipline that analyses the origin, the environment, and scope of neutralities, as well as their interactions other various ideational spectra. The central tenet of neutrosophy is that each thought contains not simply a level of actuality, as it is usually acknowledged in many-valued logic settings, as well as a degree of untruth and indeterminacy, which requires being assessed individually.

Neutrosophic reasoning originated to express mathematical patterns of unpredictability, in consistency repetition and irregularity. Neutrosophic logic is a framework for calculation truth, uncertainty, and falsehood. In the neutrophic system, uncertainty is clearly defined, and true membership, uncertainty membership, and spurious membership are all independent. This presumption is critical in a variety of scenarios, including data aggregation, that involves combining data from multiple sensors.

The neutrosophic appears and finds a place in study because everything in the world is uncertain. To begin, the primary premise of neutrosophic theory is the concept that each concept involves not just a certain level of truth, which is frequently assumed in numerous logical settings, as well as a degree of untruth and doubt that must be studied separately. As a result, in 1995, Smarandache [4] established the NS and the level of unpredictability as distinct parts for the first time. The NS $\mathbb{A}$ in $\mathbb{Y}$ is defined by: true member $T_{\mathbb{A}}(y)$, indefinite member $I_{\mathbb{A}}(y)$ and false member $F_{\mathbb{A}}(y)$.

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$T_{\mathbb{A}}(y)$, $I_{\mathbb{A}}(y)$ and $F_{\mathbb{A}}(y)$ are all true standard or non-standard section of [0,1]. This is $T_{\mathbb{A}}(y)$, $I_{\mathbb{A}}(y)$ and $F_{\mathbb{A}}(y)$: $\mathbb{Y} \rightarrow [0,1]$.

Numerous investigations in neutrosophic sets have been conducted. The idea of an FS was first introduced by Zadeh [5].Rather of having a binary membership, an FS allows things to have membership degrees ranging from 0 to 1. Many variations and extensions exist for the FS theory. Among these extensions is the notion of an intuitionistic fuzzy set (IFS) [6], which incorporates a second degree of belongingness to indicate nonbelongingness. A third function is used by another form, Picture fuzzy set, to explain the elements' neutral belongingness degree [7]. The concept of spherical belongingness functions was used by the 2019-proposed spherical fuzzy set [8] to express the degrees of belongingness of elements. Lastly, Smarandache [4] introduced the concept of neutrosophic structure, creating new avenues for exploration in the fields of decision-making and other areas.

Limitations on neutrosophic research and potential solutions:
Neutrosophic topological space can be applied to many engineering problems. Without a doubt, addressing real-world applications and developing the discipline need talking about the difficulties encountered in neutrosophic research and suggesting viable answers. The following are some major issues and possible fixes:

1) **Complexity in Analysis and Modeling:**
   Challenge: Indeterminacy is one of the extra degrees of freedom introduced by neutrophilic models, which can make modeling and analytic procedures more difficult. Solution: To reduce complexity while maintaining vital properties, create simplified representations or approximations of neutrosophic sets. Furthermore, concentrate on creating effective computational methods and algorithms for neutrosophic data.
2) **Obtaining and Verifying Data:**
   Challenge: The imprecise, partial, or contradictory character of neutrosophic data makes it difficult to obtain and validate.
   Solution: Use reliable data gathering techniques that take into consideration the ambiguity and uncertainty included in real-world data. Provide validation strategies, including approaches for managing outliers and inconsistencies, that evaluate the consistency and dependability of neutrosophic datasets.
3) **Combination with Current Approaches:**
   Challenge: It can be difficult to integrate neutrosophic sets with current mathematical models and techniques, such as probability theory and fuzzy logic.
   Solution: Investigate hybrid strategies that, in order to meet certain application needs, combine the advantages of neutrosophic sets with alternative mathematical models. Provide software libraries and standards for interoperability to make it easier to integrate neutrosophic techniques with current platforms and tools.

Through tackling these issues and putting possible solutions into practice, the field of neutrosophic research can get beyond obstacles and fulfill its potential to offer insightful analysis and solutions across a range of fields.

2. Case study

Certain parameters cannot be adequately represented by fuzzy sets. To address this shortcoming, intuitionistic fuzzy sets were developed. To further illustrate the indeterminacy, neutrosophic sets were introduced. Smarandache [4] introduced the concept of neutrosophic structure. Neutrophic sets have demonstrated their utility in resolving complex issues across multiple fields, such as robotics, medical diagnosis, image processing, pattern recognition, and decision-making. Discover the groundbreaking work of Masooma Raza Hashmi on the m-polar neutrosophic set (MPNS) and its topology [9,10]. With her innovative approach, she has developed three MCDM methods and assessed MPNS similarity through set-theoretic and cosine similarity measurements. Not only that, but her language is easy to understand, and her methods streamline the decision-making process. Join us in embracing this state-of-the-art technology today.

3. Preliminaries

In this part, we will review the basic definitions of FS and NS.

Definition 3.1. [1]
Assume $\mathbb{Y}$ is not an empty set. A FS $\mathbb{A}$ is derived from $\mathbb{A} = \{\langle y, \zeta_{\mathcal{A}}(y)\rangle : y \in \mathbb{Y}\}$, where $\zeta_{\mathbb{A}}(y): \mathbb{Y} \rightarrow [0,1]$ represents the belongingness for the FS $\mathbb{A}$. FS is a group of things with varying degrees of belongingness.

Example 3.2.
- Let $\mathbb{Y} = \{y_1, y_2, y_3, y_4, y_5\}$ be the reference set of students.
- Let $\mathbb{A}$ be the fuzzy set of "smart" students, where "smart" is fuzzy term.

$\zeta_{\mathbb{A}}(y)$'= {(y_1,0.4)(y_2,0.5)(y_3, 1)(y_4,0.9)(y_5,0.8)}
Here $\mathbb{A}$ indicates that the smartness of y_1 is 0.4 and so on.

Definition 3.3. [2]
Assume $\mathbb{Y}$ is a nonempty set. An IFS $\mathbb{A}$ in $\mathbb{Y}$ is an object with the form $\mathbb{A} = \{\langle y, \zeta_{\mathbb{A}}(y), \xi_{\mathbb{A}}(y)\rangle : y \in \mathbb{Y}\}$, where the functions $\zeta_{\mathbb{A}}(y), \xi_{\mathbb{A}}(y): \mathbb{Y} \rightarrow [0,1]$ describe the degree of belongingness and degree of non-belongingness of the element $y \in \mathbb{Y}$ within the set $\mathbb{A}$, that is a subset of $\mathbb{Y}$, and for each component $y \in \mathbb{Y}$, $0 \leq \zeta_{\mathbb{A}}(y) + \xi_{\mathbb{A}}(y) \leq 1$.

Definition 3.4. [5]
Assuming that $\mathbb{Y}$ is not an empty set, then the set $\mathbb{A} = \{\langle \zeta_{\mathbb{A}}(y), \xi_{\mathbb{A}}(y), \pi_{\mathcal{A}}(y)\rangle : y \in \mathbb{Y}\}$is called a NS on $\mathbb{Y}$, where $0 \leq \zeta_{\mathbb{A}}(y) + \xi_{\mathbb{A}}(y) + \pi_{\mathcal{A}}(y) \leq 3$ for all $y \in \mathbb{Y}$, $\zeta_{\mathbb{A}}(y), \xi_{\mathbb{A}}(y) and\ \pi_{\mathcal{A}}(y): \mathbb{Y} \rightarrow [0,1]$ are the degree of belongingness ($\zeta_{\mathbb{A}}$) the degree of non belongingness ($\xi_{\mathbb{A}}$) and the degree of indeterminacy ($\pi_{\mathcal{A}}$) of every $y \in \mathbb{Y}$ to the set $\mathbb{A}$ as well.

Example 3.5.
A mobile phone received 100 calls yesterday. Out of these, 50 were received, 35 were missed, and 15 were intentionally rejected. The degree of belongingness is 0.5, degree of

indeterminacy is 0.35, and degree of non belongingness is 0.15.

Definition 3.6. [11]
A set that is not empty NT is a subset of $\mathbb{Y}$ that meets these three axioms:
i. $0_N, 1_N \in \tau$
ii. $\mathcal{A}_1 \cap \mathcal{A}_2 \in \tau$, for $A_1, \mathcal{A}_2 \in \tau$
iii. $\cup \mathcal{A}_i \in \tau$ for all $i \in J$ *and* $\mathcal{A}_i \in \tau$

The combination of two ($\mathbb{Y}$,τ) is referred to as NTS. The members of τ are Neutrosophic open sets. A Neutrosophic set K is considered to be Neutrosophic closed if $K^c \in \tau$. K^c denotes of all Neutrosophic closed sets in this NTS.

Example 3.7.
Let $\mathbb{Y} = \{r, s, t\}$ and $\tau = \{0_N, \mathbb{P}, \mathbb{Q}, \mathbb{R}, \mathbb{T}, 1_N\}$is a collection of neutrosophic sets on $\mathbb{Y}$, where $0_N = (0,0,1)$ and $1_N = (1,1,0)$

$\mathbb{P} = \{\langle r, 0.52, 0.63, 0.47\rangle, \langle s, 0.81, 0.62, 0.47\rangle, \langle t, 0.41, 0.52, 0.77\rangle\}$,

$\mathbb{Q} = \{\langle r, 0.88, 0.75, 0.38\rangle, \langle s, 0.93, 0.62, 0.28\rangle, \langle t, 0.71, 0.62, 0.47\rangle\}$,

$\mathbb{R} = \{\langle r, 0.69, 0.75, 0.47\rangle, \langle s, 0.81, 0.67, 0.45\rangle, \langle t, 0.7, 0.61, 0.58\rangle\}$,

$\mathcal{D} = \{\langle r, 0.91, 0.82, 0.29\rangle, \langle s, 0.94, 0.78, 0.18\rangle, \langle t, 0.89, 0.77, 0.42\rangle\}$.

Clearly τ is an NT,

Then the family $(\mathbb{Y}, \tau) = \{0_N, \mathbb{P}, \mathbb{Q}, \mathbb{R}, \mathbb{T}, 1_N\}$ of NTS in $\mathbb{Y}$ implies NT on $\mathbb{Y}$. Each set in τ is an open set, with complement that are closed sets.

Definition 3.8. [12]
Because it is a parametric family of specified sections of the set NS(U), the NSS may be expressed as a set of ordered pairs $N = \{(e, \{< t, T_{f_N}(e)(t), I_{f_N}(e)(t), F_{f_N}(e)(t) > : t \in U\}) : e \in E\}$ where $T_{f_N}(e)(t), I_{f_N}(e)(t), F_{f_N}(e)(t) \in [0,1]$, accordingly called the truth-membership, indeterminacy-membership, falsity-membership function of $f_N(e)$. The inequality $0 \le T_{f_N}(e)(t) + I_{f_N}(e)(t) + F_{f_N}(e)(t) \le 3$ is self-evident since the supreme value for every T, I, F *is* 1.

Definition 3.9. [13]
Let $NSS\,(U, \mathcal{F})$ be the collection of all NSS over U with variables in $\mathcal{F}$, and $\tau_u \subset NSS\,(U, \mathcal{F})$. If the following requirements are met, τ_u is termed NST on $(U, \mathcal{F})$. 0_u, $1_u \in \tau_u$
i. $A_{u_1} \cap A_{u_2} \in \tau_u$, *for* $A_{u_1}, A_{u_2} \in \tau_u$
ii. $\cup A_{u_i} \in \tau_u$ *for all* $i \in J$ *and* $A_{u_i} \in \tau_u$

The triplet (U, E, τ_u) is then referred to as a NST space. Each member of τ_u is called a τ_u-open NSS. If the other side of an NSS is said to be τ_u closed.

Definition 3.10. [14]
Consider $(\rho, \mathcal{F})$ *to* be a neutrosophic soft cubic set (NSCS) over U.

i. If $\rho(e) = \check{1} = \{(\check{1}, \check{1}, \check{1}), (1,1,1)\} = U$ for each $e \in \mathcal{F}$, then $(\rho, \mathcal{F})$ is said to be an absolute or universal NSCS over U.
ii. If $\rho(e) = \check{0} = \{(\check{0}, \check{0}, \check{0}), (0,0,0)\} = \varphi$ for each $e \in \mathcal{F}$, then $(\rho, \mathcal{F})$ is referred to as the null or empty NSCS U.

Of course, $\varphi^c = U$ and $U^c = \varphi$.

Definition 3.11. [15]
Assume that $(\mathbb{Y}, \mathcal{F})$is a component of the NSCS $(U, \mathcal{F})$, and that $\xi(\mathbb{Y}, \mathcal{F})$ is the collection of all NSC subsets of $(\mathbb{Y}, \mathcal{F})$. If the criteria that follow are satisfied, a subfamily τ of $\xi(\mathbb{Y}, \mathcal{F})$ is known as NSCT on $(\mathbb{Y}, \mathcal{F})$.

(i) $(\varphi, \mathcal{F}), (\mathcal{X}, \mathcal{F}) \in \tau$.
(ii) $(\mathcal{P}, \mathcal{F}), (\mathcal{Q}, \mathcal{F}) \in \tau \Rightarrow (\mathcal{P}, \mathcal{F}) \cap (\mathcal{Q}, \mathcal{F}) \in \tau$.
(iii) $\{(\mathcal{P}_\alpha, \mathcal{F});\ \alpha \in \Gamma\} \in \tau \Rightarrow S\{(\mathcal{P}_\alpha, \mathcal{F});\ \alpha \in \Gamma\} \in \tau$

A NSCTS over $(\mathbb{Y}, \mathcal{F})$ is the name given to the triplet $(\mathbb{Y}, \tau, \mathcal{F})$. In $(\mathbb{Y}, \mathcal{F})$, every member of is referred to as a NSC open set.

Definition 3.12. [16]
M_R is an object in the references set Q is termed m-polar NS (mPNS), if it may be written as $M_R = \{(\omega, \langle \mu_\alpha(\omega), \sigma_\alpha(\omega), \pi_\alpha(\omega)\rangle) ;\ \omega \in Q, \alpha = 1,2,3 \dots\dots\dots m\}$ *where* $\mu_\alpha(\omega), \sigma_\alpha(\omega)$and $\pi_\alpha(\omega) : X \to [0,1]$ and $0 \le \mu_\alpha(\omega) + \sigma_\alpha(\omega) + \pi_\alpha(\omega) \le 3, \alpha = 1, 2, 3 \dots\dots\dots m$. This circumstance indicates that the three grades are $\mu_\alpha(\omega), \sigma_\alpha(\omega)$ *and* $\pi_\alpha(\omega); (\alpha = 1,2,3 \dots\dots\dots m)$ are independent and express, respectively, the object or alternative's truth, indeterminacy, and falsehood for many criteria. A straightforward representation of an m-polar neutrosophic number (mPNN) is $\tau = (\langle \mu_\alpha(\omega), \sigma_\alpha(\omega), \pi_\alpha(\omega)\rangle)$ where $0 \le \mu_\alpha(\omega) + \sigma_\alpha(\omega) + \pi_\alpha(\omega) \le 3, \alpha = 1,2,3 \dots\dots\dots m$.

Definition 3.13. [17]
Let mPN (Q) represent the grouping of all mPNSs in Q, and the non-empty referral set is represented by Q. A set of all T_{M_R} containing mPNSs meets the following criteria, it is referred to as having mPNT:

i. $0_{M_R}, 1_{M_R} \in T_{M_R}$.
ii. *If* $(M_R)_\rho \in T_{M_R}\ \forall \rho \in \Delta$ *then* $\bigcup_{\rho \in \Delta}(M_R)_\rho \in T_{M_R}$.
iii. *If* $M_{R_1}, M_{R_2} \in T_{M_R}$ then $M_{R_1} \cap M_{R_2} \in T_{M_R}$

then the pair (Q, T_{M_R}) is referred to be mPNTS. The open mPNSs are the members of T_{M_R}, while the closed mPNSs are their complements.

4. Advancement in NS and NTS

4.1. NS and NTS [11]

Zadeh [1] proposed the fuzzy set in 1965, with each element having a degree of membership. K. In 1983, Atanassov [2, 3, 19] proposed the IFS on a universe $\mathcal{X}$ represents a generalisation of FS, leaving away the degree of membership and non-membership of every element. Following the introduction of the NS notion [5,20]. There has been a lot of attention in neuosophic algebraic structures over the past few years. A formal framework that aims to quantify truth, indeterminacy, and lies is neutrosophic logic. Smarandache distinguishes NL from IFL, as well as the associated neutrosophic and IFS. In this work, A.A.Salama [11] discusses about the NS using many results and examples.Then obtained numerous features and analysed the link between neutrosophic sets and others after providing the essential definitions of neutrosophic set operations. Finally, the author apply the ideas of fuzzy topological space [21, 22] and intuitionistic fuzzy topological space [22, 23] to neutrosophic sets. The potential applications to superstrings and space-time are discussed. After that A. A. Salama [24] has discussed Generalized NS and Generalized NTS.

4.2. Neutrosophic Pre-open sets and Pre-closed Sets in NT [25]

P. Iswarya et al. [3] presented the ideas of neutrosophic semi-interior, neutrosophic semi-open sets, neutrosophic semi-closure, neutrosophic semi-closed sets, and in NTS in 2016. The notions of neutrosophic semi-interior, neutrosophic semi-open sets, neutrosophic semi-closure and neutrosophic semi-closed sets NTS were further expanded by the author. They were initially proposed in 2016 by P. Iswarya et al. [3]. This study by V. Venkateswara Rao [25] analyses some of its essential properties in NTS using examples.

4.3. NSCS in TS [15]

Many studies are being conducted in the field of topological space. In 1999, Molodtsov [26] introduced the concept of soft set theory, that is a completely novel method on representing different types of ambiguity and vagueness in situations from the real world. Soft set theory holds significant ability to infiltrate within a variety of domains, as demonstrated by Molodtsov's pioneer work [26]. The cubic set concept was established by Y.B.Jun [27] by merging fuzzy sets with period valued FS. After extending this concept to a neutrosophic context, Y.B.Jun [28] termed it NCS. Wang [29] defined and investigated the properties of the interval NS. Chinnadurai [14] investigated several characterizations of NCSS behaviors. Pramanik et al. [30] followed him, R. Anitha Cruz , F. Nirmala Irudayam [15]introducing additional operations and bringing some of the features of NSCS. This paper focuses on the systematic examination of NSCS and the derivation of various characteristics created by them. This allows us to present some comparative characterizations and emphasize their interrelationships. This study provides the framework for future research utilizing these sets to investigate alternative separation axioms. Various mappings can also be utilized to investigate various types of NSCTS connections.

4.4. Neutrosophic Crisp (NC) Bi-Topological Spaces(NCBTS)[31]

Smarandache [18, 32] introduced the term "Neutrosophy" to describe a new discipline of philosophy. Smarandache [18,32,33,34] defined NS from neutrosophy. T, F and I are three distinct elements that represent the values of true membership, false membership and indeterminacy correspondingly. T, F, and I are calculated using the uncommon unit interval [0,1]. Smarandache [18, 32] laid the groundwork for NL [4, 19], a generalisation of FL. Salama presented the Neutrosophic Crisp (NC) set Theory [35].

Alblowi [36,37] explored NS and coined the words normal NS, α-cut, convex set, and neutrosophic principles. Hanafy, Salama, and Mahfouz [38] investigated various alternative definitions for the NC Data's core principles. Some of the qualities of NTS were identified by Salama and Alblowi [11]. Salama and Alblowi [24] defined generalised NS and generalised NTS Salama and Alblowi [24] discovered generalised NTS traits in the same investigation. Salama and Elagamy [39] established filters on NS and studied a variety of correlations between neutrosophic filters and NT. Salama, Smarandache [40] looked at the connections between several neutrosophic sharp filters and NT. Salama [35] expanded the idea of crisp TS to include neutrosophic crisp TS. In the same work, Salama [35] established NC continuous function and NC compact spaces. In this paper, Riad Khidr Al-Hamido[31] presents NCBTS as a generalisation of NCTS. As novel types of open and closed sets, NC bi-open sets, NC bi-closed sets, NC S-open sets, and NC S-closed sets are introduced. We investigate the features of four unique forms of NC sets. This paper introduces the closure and inner NC set, as well as a novel concept of open and closed sets. The key characteristics and properties of these open and closed sets are investigated.

4.5. DM via NSSTS [41]

Nguyen [42] pioneered a novel notion in soft computing known as the support-neutrosophic set. Deli [43] developed the interval-valued neutrosophic soft set as a generalised idea. The author creates the interval-valued neutrosophic support soft set by combining interval-valued neutrosophic soft sets with support sets and investigates some of its essential functionalities in this work. The major goal of this study is to make judgements utilising NSSTS with interval values. In this study, Ahmed [44] investigates the concept of interval NS and proposes the creation of a new form of set in TS known as the interval valued NSS set. The main objective of this article[41] is to provide the best decision-making solution in real-world scenarios by utilising an interval valued NSS set. Parimala Mani [41] developed the interval-valued neutrosophic soft set as a generalised idea Parimala Mani presents a technique that uses an interval valued neutrosophic support soft set to discover the optimal cancer treatment under certain limitations based on a real-world issue.

4.6. Pythagorean m-polar Fuzzy Neutrosophic Topology(PmPFNTS) with Applications[17]

Smarandache expanded Neutrosophic set to Plithogenic set [45], characterised by an organisation in which each component is represented by numerous attribute values, each of which has either a fuzzy, IF, or neutrosophic degree of appurtenance to the set. Chen [46] expanded the notion of bipolar FS to m-polar FS and demonstrated various real-world applications. In 2019, Naeem [47] investigated PmPFS along with certain of its key characteristics. They additionally illustrated the application of PmPFTS in the classic technique TOPSIS in the decision-making task of picking the most appropriate form of marketing.

Atiqa Siraj's [48] major purpose in this article is to examine different features of PmPFNS and establish topology on them as well. There are several scenarios in which data consists of multi-polar figures and data. One of the better approaches for dealing with such problems is PmPFNS. It may be used to demonstrate ambiguous facts more fully and clearly. It is still utilised in a variety of applications, including combination operators, data measurements, and DM. As a consequence of this advancement, the author provides an overview of PmPFNS in order to provide an understandable viewpoint on the principles, various tools, and trends associated with their growth. The remainder of the article is organised as follows: Section 2 discusses key concepts. Section 3 delves into PmPFNS. Section 4 outlines the topological framework and essential aspects of our proposed model. The last part includes two uses for DM power. In Atiqa Siraj's [41] work, many PmPFNS traits are examined and topology is established on them. The cases when the data comprises of multipolar figures and data are covered in the article. Atiqa Siraj gave a case study and an example of using TOPSIS to make decisions in practical settings. For ease of comprehension, the article includes flowcharts and an algorithm. Furthermore, a 3D bar chart is demonstrated as a useful tool for successfully contrasting several options. By grading possibilities according to degrees of truth, falsity, and uncertainty, neutrosophic set theory facilitates decision-making.

4.7. mPNT with Applications to Multicriteria DM (MCDM) in Medical Diagnosis and Clustering Analysis[9]

Chen [46] presented the mPFS idea in 2014. First, build new notions of mPNS and topological structure on mPNS by integrating them in this research. The m-polar fuzzy soft sets with TOPSIS method for MCGDM are explained by Riaz [49]. The first to present MCDM concerns in a fuzzy setting were Bellman and Zadeh (1970)[5]. The process of MCDM Explicitly examines the best alternative(s) from the available possibilities. In the past, decisions were taken without taking into account the unknowns in the evidence, which might result in inadequate outcomes in real-world operational circumstances. If Masooma Raza Hashmi, Muhammad Riaz, and Florentin Smarandache [9] collect data without considering hesitations, the outcomes will be unclear, undefined, or equivocal. MCDM is applied in real-world circumstances such as current leadership, business, healthcare diagnostics, and a number of other fields. By integrating the mPFS and NS and developed unique notions of mPNS and topological structure on mPNS. Then, using examples, examine alternative descriptions of the mPNS and determine the different ways it operates. With examples, explain the ideas of mPNT and describe the exterior, frontier, closure, and interior for mPNS. Later, the author created a cosine equivalent measure and the set of theory resemblance scale for mPNS. Furthermore, the author proposes three strategies for MCDM in medical diagnosis and clustering analysis under unpredictability using mPNS and mPNT. Finally, the author discuss the benefits, validity, flexibility, and comparability of our suggested algorithms to current methodologies. This topology is the result of current NTS research. The m-polar neutrosophic set (MPNS) and its topology are explained by Masooma Raza Hashmi [9]. Masooma Raza Hashmi creates three MCDM methods in addition to assessing MPNS similarity using set-theoretic and cosine similarity measurements. Lastly, the authour approach employs straightforward language and streamlines decision-making. Come embrace this state-of-the-art technology with us.

4.8. Topology on Ultra Neutrosophic Set (UTS) [50]

Smarandache [18] proposed the core idea of NS, that represents a generalisation to both the FS [4] and the IFS [19]. Salama et al. [51] established the notion of Neutrosophic Crisp Set (NCS) in 2014. Salama [51] then investigated Neutrosophic Crisp Topological Space (NCTS). Salama [35] expanded his research on NCS theory in 2015. El Ghawably and Salama [38] later presented the Ultra Neutrosophic Crisp Set (UNCS) in 2015. Smarandache [18] created the fundamental concept of NS, that can be considered a generalisation of both FS [18] and IFS [19]. In 2014, Alblowi [36] developed the notion of NCS. Salama next delved into NCTS [51]. Salama [52] conducted more research on NCS hypothesis in 2015. El Ghawably and Salama [53] later introduced the Ultra Neutrosophic Crisp Set (UNCS) in 2015.

4.9. Fuzzy Neutrosophic Supra Topological Spaces(FNSTS)[54]

Supra TS, supra open sets, and supra closed sets were defined by Mashhour et al [55]. ME Abd El-Monsef [54] subsequently proposed fuzzy supra TS as a natural extension of supra TS. Dogen Coker [56,57] and Bayhan pioneered the application of Fuzzy-compactness in IFTSs a few years later. Amarenra Babu [58] extends the notion of FNS into fuzzy neutrosophic supra sets, introduces FNSTS, defines fuzzy neutrosophic supra closure and interior, and investigates some of their features in this study.

4.10. Fermatean NTS (FNTS) and an Application of Neutrosophic Kano Method[59]

The most notable disciplines in which NS are researched are topology and topological spaces. The creation of NTS has began as a result of the flaws in the fuzzy and IF ideas described in [11,60]. Senapati and Yager created Fermatean FS, a unique extension of FS, in 2020 [61]. Several researchers [62-65] have conducted extensive research on Fermatean FS for MCDM issues. In [66], Ibrahim defined Fermatean fuzzy TS.Additionally, Nazmiye Gonul Bilgin [59] several NTS. To begin, he examined various existing sets and models, including the Fermatean FS [61], NS [4], NTS [11,60], and Fermatean NS [67]. These researches are required for the creation of novel concepts in this article. The second section introduces Fermatean NS and topological structure on Fermatean NS.The third part investigates the idea of Fermatean neutrosophic continuity. The fourth part shows how to use the neutrosophic Kano approach. The fifth part provides an overview of Fermatean NS and their topological structure. The end of the work is found in the final section six. Fermatean neutrosophic topology is introduced by Nazmiye Gonul Bilgin [59], who expands on the notion of Fermatean neutrosophic sets. The topic of topological data analysis for ambiguous and indeterminate information is one that is expanding quickly. This recent tendency has led to the proposal of Fermatean neutrosophic topology, an extension of Fermatean fuzzy topology and neutrosophic topology. The Neutron Scholastic Kano method is applied to an issue of decision making.

Comparative Analysis:

In the study [68] the clear picture of the comparison of fuzzy set, intuitionistic fuzzy set, Pythagorean fuzzy set, neutrosophic set , picture fuzzy set and spherical fuzzy set as Table 1 describes.

Table 1. Comparison of Fuzzy Sets

Fuzzy sets	Intuitionistic fuzzy sets	Neutrosophic sets
These sets allow components to belong to a set to variable degrees	Fuzzy sets that are intuitive present the idea of hesitation	To simultaneously capture uncertainty,

between 0 and 1, representing uncertainty through membership degrees. $0 \leq \zeta_{\mathbb{A}}(y) \leq 1$.	degrees, which stand for the degree of doubt or hesitation connected to the membership and non-membership degrees. $0 \leq \zeta_{\mathbb{A}}(y) + \xi_{\mathbb{A}}(y) \leq 1$	vagueness, and ambiguity, neutrosophic sets explicitly reflect three components: membership, non-membership, and indeterminacy degrees. $0 \leq \zeta_{\mathbb{A}}(y) + \xi_{\mathbb{A}}(y) + \pi_{\mathcal{A}}(y) \leq 3$

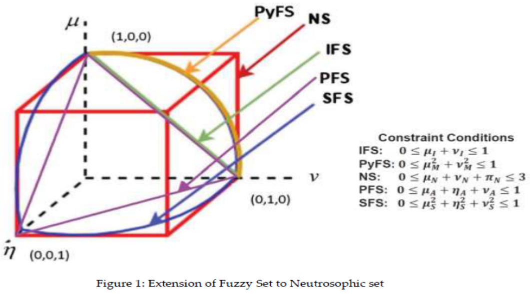

Fig. 1. Extension of Fuzzy Set to Neutrosoplic set

5. Conclusion

The membership functions for truth, indeterminacy, and falsity in NSs are independent in nature. While IFSs and FSs can only handle partial or incomplete information, NSs are far better at handling inconsistent, incomplete, and indeterminate data. Neurosophic topology (NT) and neutrosophic sets (NS) are essential tools for dealing with uncertainty in a variety of fields. This study explores the topological features extended to neutrosophic topological spaces and gives a thorough description of their applications. Since each setting is different, this review provides a condensed perspective that can be reviewed and modified later. This paper will be especially helpful to researchers studying neutrosophic topology since it provides guidance on how to investigate potential applications and provides information on appropriate methods for making decisions and how to creates three MCDM methods in addition to assessing MPNS similarity using set-theoretic and cosine similarity measurements. Furthermore, it establishes the foundation for upcoming projects that use neutrosophic set theory to analyze topological data.

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