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Investigation of Compressive Sensing and Machine Learning Techniques for Classification of Incipient Discharges in Transformer Insulation

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Abstract

Present study deals with the acquisition and analysis of different types of incipient discharges in transformer by adopting Ultra-High Frequency (UHF) technique. The Nyquist rate sampling method generates a large number of samples, making it inefficient for developing an online monitoring system. To reduce this, compressive sensing techniques are employed for signal compression and reconstruction. Various compressive sensing methods, including Convex, Non-Convex, Greedy, and Iterative Thresholding, were compared. Orthogonal Matching Pursuit (OMP) was found to be the optimal algorithm, achieving optimal reconstruction time and error at a compression ratio of 45%. The reconstructed signals were compared with the originals using Fast Fourier Transform (FFT), revealing similarities in dominant frequencies. A Long Short-Term Memory (LSTM) machine learning model was used for signal classification, consistently outperforming other algorithms. This study enhances understanding of incipient/partial discharge detection and classification, highlighting the effectiveness of innovative signal processing and machine learning approaches in power system engineering.

Keywords: Transformer, Partial Discharge, UHF signal, Classification

1. Introduction

Transformer forms the primary constituent of the power system network. The reliability of transformer in service depends mainly on the insulation structure. During operation of transformer in service, it undergoes various stresses including electrical, thermal, mechanical, and due to environmental factors. The major stresses that occur in a transformer are due to PD as a result of localized electric fields in insulation structures. Such discharges may lead to insulation breakdown and perhaps catastrophic failure of the transformer if left unattended.

Partial discharge detection and classification in transformer oil is one of the necessities of electrical power system reliability and safety. Ultra-high frequency sensors, one of the non-intrusive types of PD monitoring techniques, basically detect the electromagnetic spectrum in the range of 300 MHz to 3 GHz. Judd et al. utilized UHF sensors to measure partial discharges in power transformers in order to model the impact of electromagnetic interferences [1]. In turn, high-frequency ranges produce more samples that require greater resources for their processing and storage. Thus, a robust methodology can be taken up for efficient acquisition, compression, reconstruction, and analysis of UHF signals in PD detection and classification. Integrating such a system with machine learning algorithms will avail meaningful features. Beltle et al. studied the efficiency of UHF sensors on PD detection emanating from multiple sources within transformers and showed a periodic behavior while analyzing each discharge source singly to explain the various influences upon transformer performance [2].

UHF technique is widely used for localizing PD in systems due to its anti-interference capabilities. However, sampling UHF signals, which range from 300 MHz to 3 GHz, is challenging due to the Nyquist principle [3,4]. To overcome this, compression sensing techniques can detect and analyse UHF signals using lower sampling rates. These algorithms exploit the signal's compressibility to capture relevant information while minimizing the required sampling frequency. This allows for effective detection and analysis of UHF partial discharge signals even with sampling rates below the Nyquist threshold. Gao et al. have adopted compressive sensing technique for sampling the UHF discharge signals from PD for reducing the sampling frequencies below Nyquist rates [5].

Discharge phenomena in transformers, including as corona, surface, and internal discharges, are intricate and necessitate precise classification for fault diagnosis and maintenance. Machine learning algorithms have become potent instruments for analysing intricate data patterns related to partial discharge. These algorithms are essential for pattern recognition, feature extraction, problem diagnosis, anomaly detection, and predictive maintenance. Studies have shown the efficacy of sophisticated machine learning methodologies across several scenarios [6]. Mahidhar et al., employed Generative Adversarial Networks (GANs) to augment digital twin models of transformer insulation discharges, therefore enhancing the reliability and accuracy of data gathering [7]. Desai et al., employed Time-Frequency Transformation techniques and quadratic support vector machines to classify partial discharge sources in transformers, effectively differentiating between various discharge types based on their distinct signatures [8]. Janani et al., investigated advanced classification models such as Fuzzy Support Vector Machines (SVM), Kernel SVM, and metric multidimensional scaling, which markedly surpassed conventional methods in precisely identifying PD sources in high-voltage insulation systems [9]. Nonlinear feature extraction algorithms demonstrate

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enhanced efficacy relative to conventional methods, effectively capturing complex correlations within the data that linear approaches may neglect. The ongoing investigation and enhancement of machine learning methods in transformer defect analysis underscore their significant potential in guaranteeing the reliability and safety of high-voltage insulating systems. Integrating these sophisticated approaches would enable the field to adopt more efficient, dependable, and predictive maintenance practices, hence enhancing operational performance and minimizing the chance of transformer failures.

The integration of compressive sensing and machine learning in transformer monitoring could significantly enhance power system reliability and reduce costs. By minimizing data acquisition and processing time, these techniques enable more efficient real-time monitoring and fault detection. This not only reduces the risk of catastrophic transformer failures but also optimizes maintenance schedules, leading to lower operational costs and improved system reliability. Moreover, the reduced data storage and transmission requirements can help lower infrastructure costs for large-scale deployments

Having known the above concerns the present work is formulated on tackling the challenges posed by PD signals, which surpass the capabilities of existing resources in online monitoring systems. By employing advanced signal processing techniques assisted with machine learning algorithms to effectively monitor and classify the PD.

2. Experimental Study and Methodology

The setup for simulating partial discharge, depicted in Fig. 1, consists of three main sections: a high voltage source for generation, a defect unit with various electrode configurations, and sensors with a storage unit for detection and analysis of partial discharge.

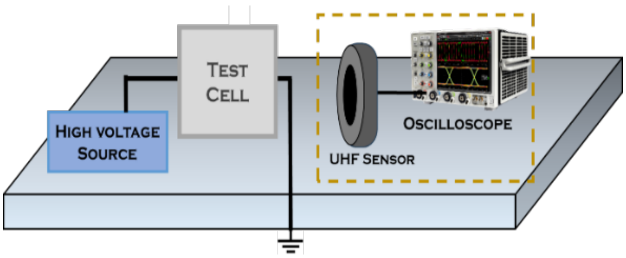

Fig. 1. Setup to simulate Partial Discharge.

Ultra-high frequency (UHF) sensors are linked to a digital storage oscilloscope (DSO) with a bandwidth of 3.5 GHz and a sampling rate of 40 GS/s, facilitating signal analysis and storage. This comprehensive setup enables the generation, detection, and analysis of partial discharge events, contributing to a deeper understanding of their characteristics and behaviours. Different configurations are utilized to simulate various types of partial discharge phenomena. Corona discharge involves a needle electrode applying high voltage to a grounded plane electrode, both made of aluminium, with a 5 mm gap distance. Surface discharge conforms to IEC 60112 standards, with electrodes set at a 60° angle above the specimen, spaced 10 mm apart. Particle discharge utilizes a 10 mm diameter spherical high voltage electrode and a curved ground electrode to cradle a copper particle. Void discharge employs a 1mm x 1mm void in pressboard between aluminium disc electrodes, immersed in transformer oil to prevent additional discharges. These setups ensure standardized testing conditions for accurate analysis, promoting understanding of partial discharge behaviour.

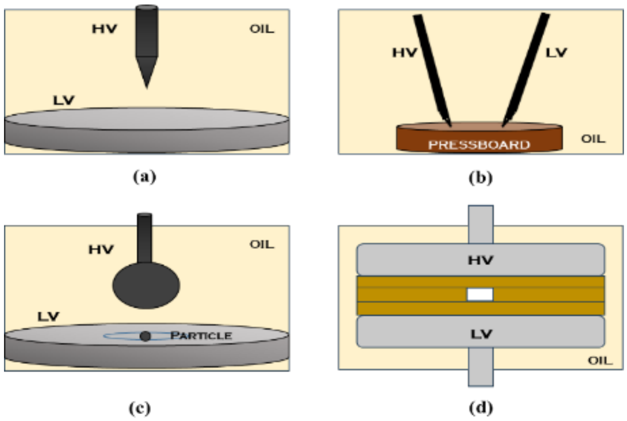

Fig. 2. Different partial discharge configuration (a) Corona discharge, (b) Surface discharge, (c) Particle discharge and (d) Void discharge.

Compressive Sensing

Compressive sensing is an important signal-processing technique for both biomedical imaging and structural health monitoring, enabling the accurate reconstruction of sparse signals from incomplete measurements. The methodologies used, such as convex optimization, greedy algorithms, iterative thresholding, and non-convex approaches, are highly effective for detecting partial discharge in transformer oil [10,11]. In power systems management, CS has emerged as a transformative strategy, particularly for real-time monitoring and control. Another team proposed a CS-based approach for load frequency control in multi-area interconnected power systems [12]. This approach significantly reduces data transmission sizes while ensuring high recovery accuracy at the central controller, thereby enhancing the reliability of communication networks, which are essential for the effective management of complex power systems with increasing load demands. In another study further demonstrated the application of CS in optimal reactive power control [13]. Their approach effectively addressed both data volume and operational efficiency challenges by minimizing data size and power loss, showcasing the practical value of CS in improving performance. In the context of smart grids, investigated the use of CS for online topology identification, reformulating the problem as one of sparse recovery [14]. Their findings suggest that CS can effectively handle data from distributed generators, thereby enhancing system security and monitoring capabilities. Additionally, highlighted the potential of CS in harmonic analysis, achieving improved frequency resolution without extending observation time [15]. In the study CS to detect harmonic frequencies in electrical systems [16]. Their work demonstrated that fewer samples can effectively reconstruct signals, showcasing CS's potential to reduce measurement overhead while maintaining accuracy. The author developed a stochastic economic dispatch algorithm using CS to handle high-dimensional uncertainties in distribution systems [17]. Their approach reduces computational costs while improving statistical accuracy, which is crucial for modern power system operations. Other team addressed supraharmonic emissions in smart grids, proposing a CS-based technique to achieve higher frequency resolution at reduced sampling rates [18]. This advancement meets the growing demand for effective monitoring of the power quality impacts of new technologies. It enables better tracking of transient changes in power quality, further demonstrating the versatility and effectiveness of compressive sensing across various engineering and technology applications in power system monitoring, operation, and control, thus enhancing network reliability

Basic Pursuit: Convex methods in compressive sensing, like Basis Pursuit, offer computationally efficient solutions for sparse signal reconstruction. To sample signals below the Nyquist rate, they compress data using a random sensing matrix which should satisfy certain property called Random Isometry Property [19]. Through linear programming, they minimize the L1 norm, ensuring accurate signal recovery from sparse measurements [20, 21]. Employing the primal-dual interior point method, these techniques converge to global optima. Newton's iteration optimizes solutions, relaxing parameters to bias towards optimal results. The process continues until the surrogate duality gap meets predefined criteria, indicating optimal reconstruction.

Iterative Hard Thresholding: Iterative thresholding algorithms form the basis for sparse signal recovery and include variants such as IHT. Such techniques are finding applications in tasks that require efficient reconstruction from highly incomplete data, for example, Compressed Sensing MRI and distributed sensor networks. IHT iteratively thresholds the signal coefficients and has achieved relatively good performance with low memory and computational cost. [22] These algorithms, in general, rely on proper definition of key elements in UHF signal analysis for partial discharge: the measurement matrix and the sparsity parameters. Sparsity is enforced by initializing with a zero vector and then using an iterative update rule involving hard thresholding until convergence, which results in a sparse estimate of the vector preserving the salient features of the input signal.

Sparse Bayesian Learning: Sparse Bayesian Learning (SBL) is a signal reconstruction algorithm that utilizes a Bayesian framework and iterative procedures [23]. Initially, it sets up input parameters, including the measurement vector (y) and forward model matrix (Φ). Hyperparameters (α) governing signal distribution are initialized, along with an estimate of the sparse signal (a). The algorithm iteratively computes statistics, updating activations and variances for signal elements. It selects the most influential element based on unspecified criteria and refines the signal estimate accordingly. This iterative process continues until convergence criteria are met, yielding the final estimate of the sparse signal vector. SBL gradually enhances signal reconstruction through successive iterations.

3. Results and Discussion

UHF Signals for Partial Discharge

UHF signals were captured for four different types of partial discharge in transformer oil. Each discharge exhibited distinctive waveform characteristics in the UHF frequency range, offering valuable insights into the underlying phenomena. Fig 3 shows UHF signal captured for various partial discharges.

Impact of Compression Ratio on Reconstruction Time

Fig. 4 displays the reconstruction error percentage for four compressive sensing models: BP, IHT, SB, and OMP. The x-axis represents compression ratio, and the y-axis shows reconstruction error. The graph illustrates the trade-off between compression ratio and reconstructed signal accuracy. A 10% error threshold was used to determine acceptable compression ratios for each model As compression ratio decreases, reconstruction error generally increases across all models due to the loss of information from the original signal. Compression removes redundant or less essential data while preserving essential features. However, as compression ratio decreases, more data is removed, resulting in greater information loss and higher reconstruction error when the signal is reconstructed.

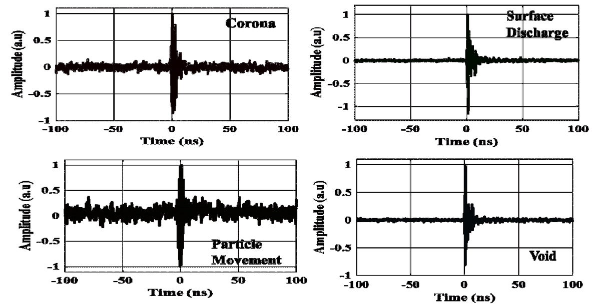

Fig. 3. UHF Signals for different partial discharge

OMP exhibits the lowest reconstruction error at lower compression ratios (around 30% and below) by selecting the best matching atoms from the dictionary to represent the signal sparsely. OMP excels in finding optimal sparse representation where signal sparsity is high, effectively capturing essential components while discarding noise and redundancy. Additionally, at lower compression ratios, OMP efficiently exploits signal sparsity and accurately reconstructs it with minimal error, as the dictionary size remains relatively large compared to the compressed signal size.

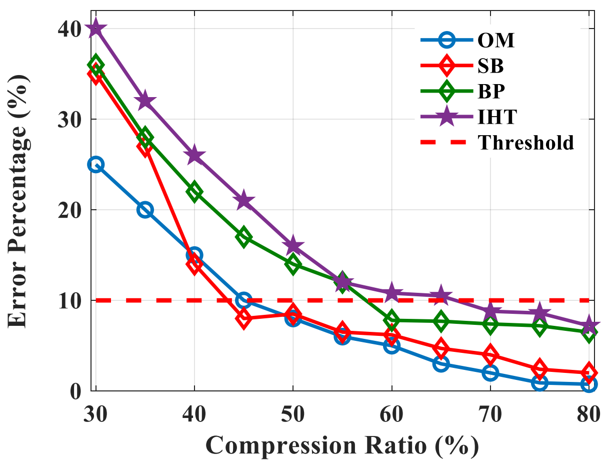

Fig. 4. Variation of reconstruction error with respect to compression ratio.

Impact of Compression Ratio on Reconstruction Time

The reconstruction time, together with accuracy, is a vital evaluation metric in compressive sensing. The relationship between reconstruction time and compression ratio is depicted in Fig. 5, demonstrating that reconstruction time tends to increase with the compression ratio, while reconstruction error typically rises at higher ratios.

Moreover, different algorithms exhibit varying reconstruction times, as shown in Fig.5. For example, out of the examined techniques, OMP demonstrates the quickest reconstruction time, highlighting the potential for accuracy and speed trade-offs. To determine the most suitable compressive sensing method for a specific application, it is essential to understand the interplay between reconstruction time and compression ratio.

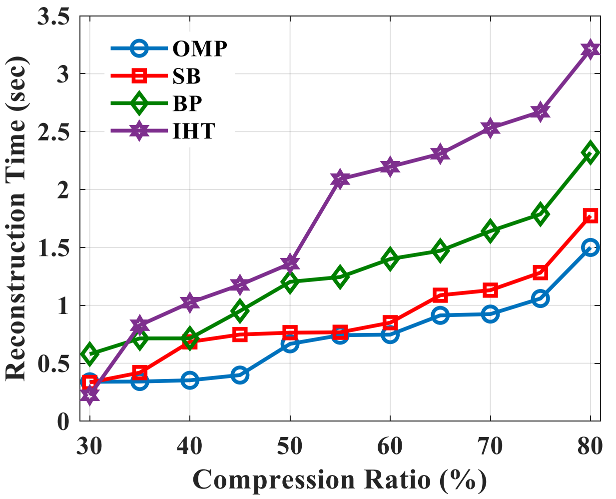

Fig. 5. Variation of Reconstruction time with respect to compression ratio

A comprehensive comparison of reconstruction error (%) and reconstruction time (s) for each of the four approaches is presented in Table 1, covering a range of compression ratios. This comparison facilitates the selection of the optimal algorithm based on the desired balance between reconstruction time and accuracy.

Fast Fourier Transform (FFT)
A detailed examination of both the original and rebuilt signals is essential to assess the performance of compressive sensing (CS) methods for signal reconstruction. The frequency content and spectral properties of the original signal are revealed by applying the Fast Fourier Transform (FFT) to both signals. The reconstruction process' fidelity is assessed using the FFT on the reconstructed signal. Reconstructed UHF waveforms with a 45% compression ratio are displayed in Fig. 6 along with the equivalent signals from four recovery algorithms and the FFT of the original signal. The quality of the reconstruction is shown by the overlap between these lines. While several approaches had considerable faults, the OMP method provided satisfactory reconstructed signals.

Table 1. Reconstruction error and Reconstruction time with various compression ratio

CR	Reconstruction Error (%)				Reconstruction Time (sec)			
	BP	IHT	SB	OMP	BP	IHT	SB	OMP
30	36	40	35	25	0.57	0.22	0.33	0.340
35	28	32	27	20	0.71	0.82	0.41	0.342
40	22	26	14	15	0.71	1.02	0.68	0.353
45	17	21	10	7.8	0.95	1.17	0.74	0.398
50	14	16	8.5	6.9	1.20	1.36	0.76	0.667
55	12	12	6.5	5.8	1.24	2.08	0.76	0.742
60	7.8	10.8	6.2	4.6	1.40	2.19	0.85	0.747
65	7.7	10.5	4.7	2.5	1.47	2.30	1.08	0.913
70	7.4	8.8	4	2	1.64	2.53	1.13	0.924
75	7.2	8.6	2.4	0.9	1.78	2.67	1.28	1.059
80	6.5	7.2	2	0.75	2.32	3.21	1.77	1.499

Table 2. Reconstruction error and Reconstruction time with various compression ratio.

Feature	Algorithms				
	Actual	BP	IHT	BP	OMP
Shape Factor	2.32	1.839992	1.78988	2.039999	2.279864
Kurtosis	54	42.8274	41.661	47.48274	53.0658
Skewness	0.68	0.539308	0.52462	0.597931	0.668236
Impulse Factor	25	19.8275	19.2875	21.98275	24.5675
Crest Factor	11.1	8.80341	8.56365	9.760341	10.90797
Clearance Factor	35.2	27.91712	27.1568	30.95171	34.59104

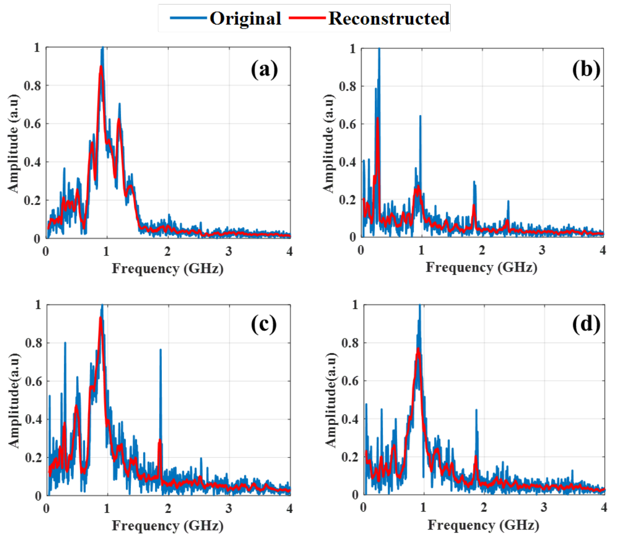

Fig. 6. FFT of Original Signal and Reconstructed Signal of OMP at 45% compression ratio (a) Corona, (b) Surface, (c) Particle and (d) Void discharge.

Impact of Reconstruction on extracted features
The study compares features like peak value, crest factor, clearance factor, skewness, kurtosis, and impulse factor in signal analysis and PD classification. Table 2 shows that the reconstruction process significantly impacts the preservation of essential characteristics. The results show that certain algorithms, like OMP, exhibit a closer resemblance to the original signal values, suggesting OMP excels in preserving essential characteristics for PD classification during the reconstruction process.

The Orthogonal Matching Pursuit (OMP) algorithm is proficient at maintaining essential signal characteristics, including skewness, kurtosis, crest factor, and impulse factor, throughout compression and reconstruction phases. These attributes are essential for preserving signal integrity, particularly in predictive diagnostics (PD) classification. The OMP approach consistently exhibits low distortion, preserving the signal's intrinsic features and enabling precise analysis and interpretation. The superior performance of the Long Short-Term Memory (LSTM) model in predictive diagnostics directly correlates with the quality of signal information retained by the OMP algorithm. The integration of OMP for compression and LSTM for classification establishes a robust framework for accurate predictive diagnostics, hence improving decision-making across diverse applications.

Machine Learning based Classification
To give the performance benchmark of the LSTM model, several of the most widely-used machine learning models are compared to it in MATLAB: SVM, kNN, Random Forest, and Decision Tree. These models will be trained using the same set of features extracted from reconstructed signals, and then their classification will be given. The project deals with a comprehensive evaluation aimed at comparing different machine learning approaches in the classification of PD types using reconstructed signal data by finding their relative strengths and weaknesses. Various algorithms are considered in this work, from simple, traditional classifiers like Support

Vector Machine to complex methods such as Random Forest. The LSTM model performed as the most capable classifier, which gave an accuracy of 99% in perfect classification. Other models, namely Decision Tree, kNN, Random Forest, and SVM, also showed very promising results; however, the LSTM outperformed these and gave the best accuracy among other techniques shown in Fig. 7. For instance, Decision Tree gave an accuracy of 79.6%, kNN and Random Forest highlighted 85.42% and 89.5%, respectively, while SVM highlighted 92.47% accuracy. For every model, the values of hyperparameters had been rather carefully tuned with grid search before any evaluation to be optimally set for accurate classification. The LSTM-specific hyperparameters were tuned and are presented in Table 3, showing our effort of thorough optimization for improved performance of the model.

Table 3. Hyperparameters for LSTM model.

Hyperparameters	Value
Activation Function	Re-Lu
Loss Function	Sparse Categorical Cross Entropy
Optimizer	Adam
Learning Rate	0.001
Epsilon	1e-08
Decay Rate	0.004
Epochs	50
Dropout	0.1

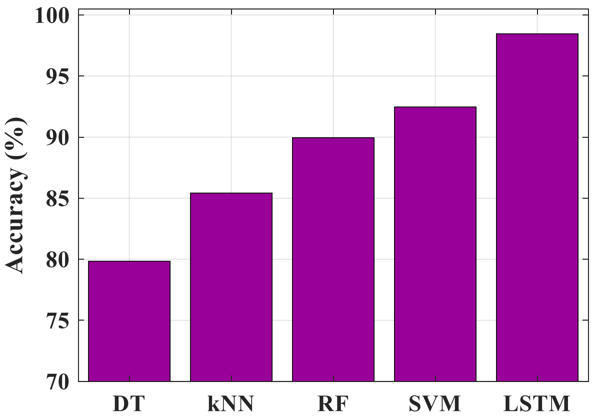

Fig. 7. Performance of various machine learning models in terms of accuracy.

The application of compressive sensing and machine learning in transformer monitoring can result in substantial enhancements in power system reliability and cost efficiency. These techniques facilitate more efficient real-time monitoring and fault detection by minimizing data collecting and processing time. This method not only mitigates the possibility of severe transformer failures but also streamlines maintenance schedules, lowering operational expenses and improving system reliability. Moreover, the diminishment of data storage and transmission necessities can decrease infrastructure expenses for extensive implementations.

4. Conclusion

Reconstruction and investigation of the ultra-high frequency signal generated by partial discharges in different types of transformer oil and paper insulation systems, such as corona discharges, surface discharges, particle movement, and void discharges, are performed through this work. Key findings: The captured UHF signals had main frequency content from 300 MHz through 3 GHz, with the dominant concentration around 1 GHz, with a temporal duration of roughly 50 nanoseconds; several CS methods were assessed in terms of signal compression and reconstruction—Convex, Non-Convex, Greedy, and Iterative Thresholding methods. The results indicated a trade-off between the compression ratio and reconstruction accuracy; higher compression ratios meant faster reconstruction times, but with increased errors. By choosing a compression ratio of 45%, the efficiency can be well balanced with fidelity, targeting less than 10% of maximum reconstruction error for signal accuracy to further provide analysis. Reliability of the reconstructed signals was further tested using a Fast Fourier Transform analysis, showing a high degree of similarity with the original signals. In addition, the proposed LSTM model has shown excellent performance in the classification of different types of PD, giving almost 0.99 R^2 in all tested samples, beating other traditional classification algorithms such as SVM, Random Forest, Decision Tree, and KNN, which generated a bit low accuracy. Integrating compressive sensing techniques with UHF sensors in real-world transformer monitoring offers significant advantages, such as reducing data transmission bandwidth and storage needs. LSTM-based classification models can be deployed in cloud or edge computing systems to facilitate real-time monitoring and diagnostics. However, challenges remain, including sensor calibration, noise interference, and signal attenuation, which must be addressed to ensure reliability.

Future research will aim to apply these techniques to other critical power system components, such as circuit breakers and cables. Additionally, the potential for real-time monitoring will be explored to enhance fault detection responsiveness and accuracy. Incorporating environmental factors like temperature, humidity, and electromagnetic interference into the experimental setup will further strengthen the robustness of these methods in real-world conditions.

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