

## Development of a Mathematical Lumped Parameters Model for the Heat Transfer Performance of a Solar Collector

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### Abstract

This work describes the developed of a lumped parameter model and demonstrates its practical application. The lumped parameter mathematical model is a useful instrument to be used for rapid determination of design dimensions and operational performance of solar collectors at the designing stage. Such model which incorporates data from relevant Computational Fluid Dynamics design and experimental investigations can provide an acceptable accuracy in predictions and can be used as an effective design tool. A computer algorithm validates the lumped parameter model via a window environment program.

*Keywords:* Lumped parameters mathematical model, flat plate collectors, Solar energy, Enhanced heat transfer, Partial metal porous medium.

### 1. Introduction

In the 60's a method in order to estimate the performance of solar water heaters circulating to a storage tank by thermosyphon was investigated. Two absorber and tank systems were tested and the results compared with those estimated from the theoretical method developed by a computer model. This method incorporated a collector efficiency factor, the solar radiation and the air temperature as a Fourier series in time [1]. In the mid 70's more comprehensive studies to evaluate the thermal performance of a thermosyphon system were conducted [2, 3]. In the 80's the long term performance, transient response, system modelling and operation characteristics of a thermosyphon system with vertical or horizontal storage tank were investigated. Its performance was maximised when the daily collector volumetric flow was approximately equal to the daily load flow [4].

A numerical and an experimental investigation of the flow and temperature distribution in a solar collector panel with an absorber consisting of parallel connected horizontal fins were performed. Corrections were determined based on the difference between the measured solar collector fluid outlet temperature and the mean of the measured absorber tube wall temperatures [5]. A similar study described above, was developed using a 3-D mathematical model for solar flat plate collectors based on setting mass and energy balances

on finite volumes. The model was validated experimentally with a commercial solar water collector.

Generally external disturbances that are independent of each other can affect an experimental process. Therefore researchers, while conducting an experiment, do not measure a real value each time but usually observe a random quantity. Hence the mathematical model is made by averaging the measured results [6, 7].

Generally the flow distribution through the collector's finned tubes clearly affects the operational efficiency of the collector system, Jianhua [8]. Therefore, the more uniform the flow is through the tubes, the efficiency of the collector is higher, and vice versa, Jones and Lior [9]. One passive method that in order to uniform the flow and enhance the heat transfer between the working liquid and the metal part of the collector is the use of a metal porous medium placed in channels of heat exchangers, Nield and Bejan [10], Baytas and Pop [11]. The presence of a metal porous medium (i.e. copper, aluminium) inside a pipe causes a better thermal dispersion and also increases the interface between the fluid and the absorber.

### 2. Lumped Parameters Model Development

The energy conservation equation is the governing equation in the lumped parameter model. Considering Figure 1, the energy of the working fluid entering the pipe at a distance  $y$ , plus the useful gain  $q'_u$  of the tube and fin will be equal to energy gained at distance  $y+\Delta y$ .

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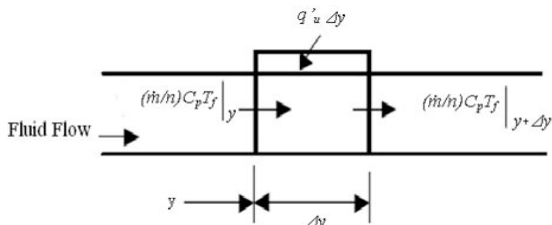


Fig. 1. Energy balance on fluid element.

In open literature there is no a general analytical equation for calculation of the collector efficiency factor  $F'$  as function of heat transfer coefficient and the overall heat losses coefficient.

This dependence usually presented in the form of tables or charts. The tabulated data from the literature was used to derive a general equation in the form of  $F'=f(h_f, U_L)$ .

An example of the tabulated data is shown in Table 2a. Combination of three values of the heat transfer coefficient,  $h_{fi}$  (100, 300 and 1000  $W/m^2C$ ) and the collector overall loss coefficient  $U_L$  (2, 4 and 8  $W/m^2C$ ) were used to derive the general equation that could cover a wide range of flow regimes (laminar and turbulent). Data listed on Table 2a present results taken from Duffie and Beckman.

Tab. 2a. Data of  $h_f, U_L$  and  $F'$

| $h_f (W/m^2C)$ | $U_L (W/m^2C)$ | $F'$  |
|----------------|----------------|-------|
| 100            | 2              | 0.930 |
| 100            | 4              | 0.865 |
| 100            | 8              | 0.765 |
| $h_f (W/m^2C)$ | $U_L (W/m^2C)$ | $F'$  |
| 300            | 2              | 0.970 |
| 300            | 4              | 0.940 |
| 300            | 8              | 0.890 |
| $h_f (W/m^2C)$ | $U_L (W/m^2C)$ | $F'$  |
| 1000           | 2              | 0.985 |
| 1000           | 4              | 0.970 |
| 1000           | 8              | 0.945 |

Five new parameters, namely  $X, Y, Z, B,$  and  $C$  were introduced to be used in the generalised equation:

$$X=7-U_L \quad (1)$$

$$Y=(9-h_{fi})/100 \quad (2)$$

$X$  and  $Y$  refer to the series of values of the heat loss coefficient  $U_L$  and the heat transfer coefficient  $h_{fi}$ , respectively.

The collector efficiency factor is given as:

$$F' = \frac{1/U_L}{W[\frac{1}{U_L[D+(W-D)]F} + \frac{1}{C_b} + \frac{1}{\pi D_i h_{fi}}]} \quad (3)$$

In order to define  $F'$  for wide range of  $h_f$  and  $U_L$  values a new generalised equation was introduced in the following form:

$$F' = Z + \sum_{i=0}^X B + i^2 \cdot C \quad (4)$$

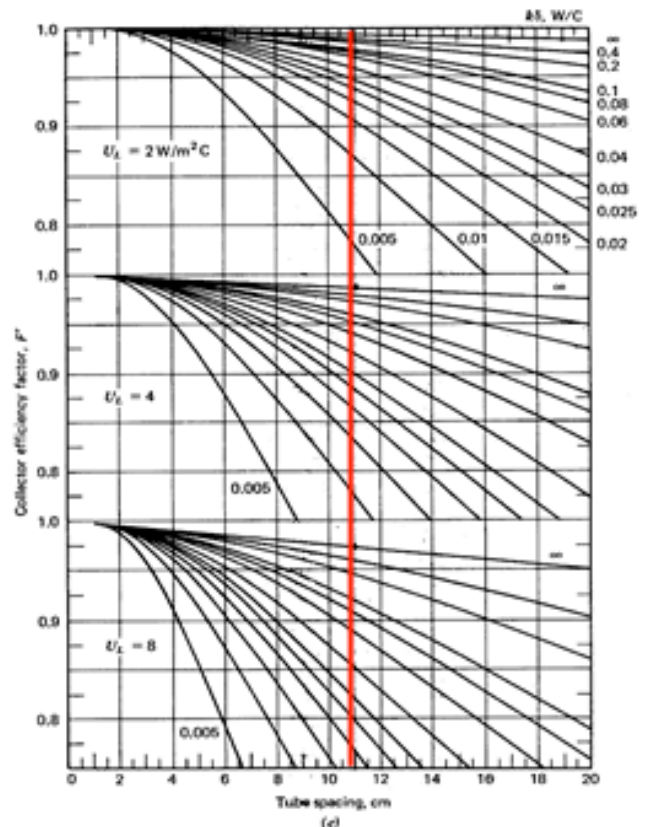
where  $Z, B,$  and  $C$  are:

$$Z = 9.45 \cdot 10^{-1} - \sum_{i=0}^X 10^{-4} + i^4 \cdot 11.66 \cdot 10^{-6} \quad (5)$$

$$B = 5.99 \cdot 10^{-3} - \sum_{j=0}^Y 10^{-5} + j^4 \cdot 11.05 \cdot 10^{-7} \quad (6)$$

$$C = 3.4 \cdot 10^{-5} - \sum_{j=0}^Y 10^{-6} + j^4 \cdot 1.8 \cdot 10^{-8} \quad (7)$$

The above equation was checked against charts as shown on Graph 1 below, presented by Duffie and Beckman which display various values of  $F'$  versus tube spacing and various overall heat loss coefficient values  $U_L$ . It was established that equation (4) can predict the value of  $F'$  without the use of charts. The vertical line indicates that the tube spacing used in the experiments and the theoretical analysis were 11cm apart.



Graph. 1.  $F'$  vs. tube spacing for 10mm diameter tubes ( $h_i=1000 W/m^2$ ).

### 3. Validation of the lumped parameter model.

A computer algorithm was created using equations 1-9 in order to cross examine the validity of the mathematical model. This algorithm was formed into a window mode program that enables the user to input variables, like the area

of the collector  $A_c$ , the overall heat loss  $U_L$  etc. The aim of the whole process was to determine the values of  $F'$  and the water output Temperature  $T_{fo}$  of the collector. Using data from Table 2 shown below the values of  $U_L=2 (W/m^2C)$  and  $h_f=1000 (W/m^2C)$  were inputted to the program as shown below on Fig.2. The result obtained is illustrated on Fig.3 where the graph clearly shows that the value of  $F'$  is around to 0.985 similar to Duffie's findings shown on Table 2a. The computer program created can produce graphs of  $F'$  vs  $h_{fi}$ ,  $T_{fo}$  vs  $h_{fi}$  and  $T_{fi}$  vs  $T_{fo}$ .



Fig. 2. Lumped parameter model command window

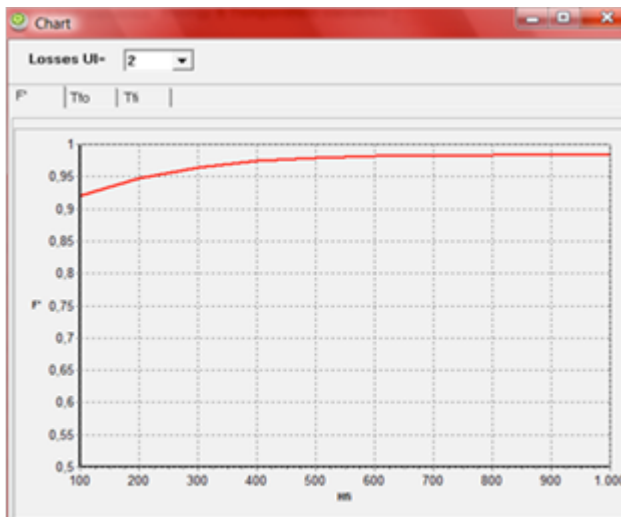


Fig. 3. Graph of  $F'$  vs  $h_{fi}$

Experimental results obtained during tests in both conventional and the collector with the metallic mesh insertion, were used to validate the developed lumped parameter model. Table 3a illustrates results from experiments (bold data) for the conventional and the porous medium collectors.

$$T_{fo} = \left( T_a + \frac{S}{U_L} \right) - \left[ -T_{fo} + T_a + \frac{S}{U_L} \right] \exp \left( \frac{A_c F' U_L}{\dot{m} C_p} \right) \quad (8)$$

Combined with data acquired from the literature, were both used to calculate theoretically the temperature of the water in the outlet part of the collector section  $T_{fo}$  using equation (8) in order to cross examine the experimental results to the theoretical techniques employed.

Tab. 3a. Data for the conventional and porous collector.

| Variables                 | Conventional Collector | Porous Collector |
|---------------------------|------------------------|------------------|
| $W$ (m)                   | 0.11                   | 0.11             |
| $D$ (m)                   | 0.01                   | 0.01             |
| $S$ ( $W/m^2$ )           | 905                    | 905              |
| $U_L$ ( $W/m^2C$ )        | 6.55                   | 6.55             |
| $T_b$ $^{\circ}C$         | <b>149.6</b>           | <b>148.7</b>     |
| $T_a$ $^{\circ}C$         | 18                     | 18               |
| $k$ ( $W/m^{\circ}C$ )    | 400                    | 400              |
| $d$ (m)                   | 0.0005                 | 0.0001           |
| $D_i$ (m)                 | 0.008                  | 0.007            |
| $m$                       | 5.7                    | 5.7              |
| $T_{fi}$ $^{\circ}C$      | <b>65.6</b>            | <b>81.7</b>      |
| $A_c$ ( $m^2$ )           | 0.64                   | 0.64             |
| $\dot{m}$ (mg/s)          | <b>0.62</b>            | <b>0.58</b>      |
| $C_p$ ( $J/Kg^{\circ}C$ ) | 4230                   | 4240             |

On Table 3a,  $W$  is the width of the fin,  $D$  is the external diameter of the pipes,  $S$  is the solar heat flux,  $U_L$  is the collector overall loss coefficient,  $T_b$  is the base temperature at the joint of fin and tube,  $T_{ref}$  is the fluid reference temperature  $T_a$  is the ambient temperature,  $k$  is the thermal conductivity of the metal,  $d$  is the thickness of the fin.  $D_i$  is the internal diameter of the pipe,  $m = \frac{\sqrt{U_L}}{\sqrt{k\delta}}$ ,  $T_{fi}$  is the

temperature of the working fluid at the inlet,  $A_c$  is the area of the collector,  $\dot{m}$  is the volumetric flow rate, and  $C_p$  is the specific heat of the working fluid. Fig. 4 shows a cross-sectional representation of a tube-fin section and points out some of the variables listed on Table 3a.

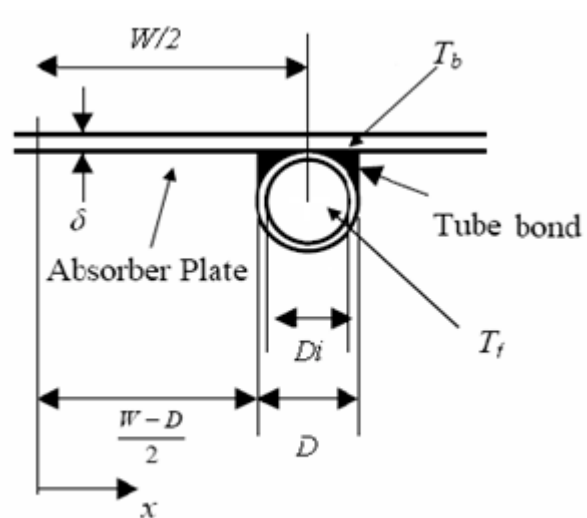


Fig. 4. Collector fin - tube section.

The function  $F$  which is the standard fin efficiency for straight fins with rectangular profile was obtained from equation (9) for both collectors having a value of **0.974**.

$$F = \frac{\tanh[m(W - D)/2]}{m(W - D)/2} \quad (9)$$

**For the Conventional Collector:**

The useful gain  $q'_u$  for the tube and fin per unit length in the flow direction was calculated using equation (10) for both copper and aluminium finned pipes having a value of **7.29**.

$$q'_u = [(W - D)F + D][S - U_L(T_b - T_a)] \quad (10)$$

The thermal resistance to heat flows from the plate to the fluid resulted from the plate-tube bond conductance and the tube-to-fluid convection heat transfer inside of tubes.

Reynolds number  $Re$  was calculated as:

$$Re = \frac{4\dot{m}}{\pi D \mu} \quad (11)$$

The dynamic viscosity ( $\mu$ ) was taken from charts for 133°C. In equation (11)  $\dot{m}$  is the total collector flow rate (0.62ml/sec),  $D$  (0.01m) is the diameter of the pipe,  $\mu$  is the dynamic viscosity ( $0.209 \cdot 10^{-3}$  Kg/ms).

The value of the  $Re$  number calculated was **377** which represented a laminar flow (since Reynolds found that for a flow in a pipe it did not matter which of the parameters  $\dot{m}$  or  $D$  varied in this dimensionless group, as long as  $Re$  was less than approximately 2300, the flow is laminar).

The collector efficiency factor  $F'$  calculated was **0.873**.  $F'$  contains the value of the local heat transfer coefficient  $h$  obtained in experiment and equal to **238** W/m<sup>2</sup>°C.

Finally, the theoretical water outlet temperature was calculated from equation (8) and was equal to **133.4** °C which was very close to the one found in the experiments (**133** °C).

**For the Porous Medium Collector:**

The useful gain  $q'_u$  for the tube and fin per unit length in the flow direction was determined for both copper and aluminium finned pipes and was equal to **7.45**.

Reynolds number  $Re$  was calculated for the water temperature of 140°C as found in the experiments. In this case the values of  $\dot{m}$  was 0.58ml/sec,  $D$  was 0.01m and  $\mu$  was  $0.197 \cdot 10^{-3}$  Kg/ms. The value of  $Re$  number calculated was **373** which corresponds to a laminar flow.

The collector efficiency factor  $F'$  obtained was **0.885**.  $F'$  was calculated for the value of the local heat transfer coefficient  $h$  equal to **266** W/m<sup>2</sup>°C (from the experiments).

The theoretical output temperature was calculated from equation (8) was **139.5** °C which was very close to the one found in the experiments (**140** °C).

From the findings so far the average  $Ra$  and  $Nu$  number for the porous and conventional cases could be obtained over a characteristic pipe length. The average  $Ra$  number could be obtained from the expression:

$$Ra = \frac{C_p \rho^2 g \beta (\Delta T) L^3}{\kappa \mu} \quad (12)$$

where  $C_p$  is the specific heat of the water,  $\rho$  is the density of the water,  $g$  is the gravitational force at (55° inclination),  $\beta$  is the thermal expansion coefficient,  $\Delta T$  is the temperature difference between the outer pipe surface and the water temperature at the pipe wall,  $L$  is the characteristic length of the pipe,  $\kappa$  is the thermal conductivity and  $\mu$  the dynamic viscosity. The average  $Ra$  number was obtained for the heat flux of 905 W/m<sup>2</sup>.

The data used to obtain the Rayleigh numbers is listed on Table 3.b. The water properties were taken from charts for the temperatures of 140°C (porous medium collector) and 133°C (conventional collector).

|                                | Conventional Collector | Porous medium Collector |
|--------------------------------|------------------------|-------------------------|
| $C_p$ (J/Kg °C)                | 4270                   | 4286                    |
| $\rho$ (Kg/m <sup>3</sup> )    | 930                    | 922                     |
| $g$ (m/s <sup>2</sup> )        | 5.62                   | 5.62                    |
| $\beta$ (1/°C)                 | $0.93 \cdot 10^{-3}$   | $0.97 \cdot 10^{-3}$    |
| $\Delta T$ (°C)                | 3.0                    | 3.2                     |
| $L$ (m)                        | 0.25                   | 0.25                    |
| $\kappa$ (W/m <sup>2</sup> °C) | $0.684 \cdot 10^{-3}$  | $0.683 \cdot 10^{-3}$   |
| $\mu$ (Kg/m sec)               | $0.208 \cdot 10^{-3}$  | $0.197 \cdot 10^{-3}$   |

Tab. 3b. Data for the  $Ra$  Numbers.

The average  $Ra$  Numbers calculated for the convectional collector and for the porous medium collector was  **$6.35 \cdot 10^9$**  and  **$7.38 \cdot 10^9$**  respectively.

The average  $Nu$  number determined in relation to  $Ra$  number for the porous medium case for a heat flux of 905 W/m<sup>2</sup> given by the expression (13):

$$Nu_{(905)} = 0.0107(Ra)^{0.285} \quad (13)$$

The  $Nu$  Number obtained for the collector utilising the metallic mesh was **6.91** and for the conventional collector was **5.75**.

To summarise all the findings, Table 3c was created and illustrates all the results for both collectors.

It can be observed that the collector with the metallic insertion material has greater values of  $F'$ ,  $T_{fout}$  and  $h_f$  which indicates the enhancement in the heat transfer in the collector.

The  $Nu$  and  $Ra$  Numbers are greater in the case of the porous medium collector supporting the enhancement of the convective heat transfer. The output temperature  $T_{fout}$ , based on the model created by Duffie and Beckman, obtained in both cases from equation (8) is approximately  $\pm 0.3$ °C accurate.

Tab. 3c. Porous medium and conventional collector parameters.

| Variables                             | Porous medium Collector | Conventional Collector  |
|---------------------------------------|-------------------------|-------------------------|
| $h_f$ (heat transfer coefficient)     | 266 W/m <sup>2</sup> °C | 238 W/m <sup>2</sup> °C |
| $F'$ (collector efficiency factor)    | 0.885                   | 0.873                   |
| $Re$ (Reynolds number)                | 373                     | 377                     |
| $T_{fout}$ (output water temperature) | 139.5 °C                | 133.4 °C                |
| $Ra$ Number                           | $7.38 \cdot 10^9$       | $6.35 \cdot 10^9$       |
| $Nu$ Number                           | 6.91                    | 5.75                    |

#### 4. Conclusion

The experimental results validated by the lumped parameter model and demonstrated that can accurately predict the performance of the collector and therefore can be used in the designing process. The computer algorithm also supported these findings.

The average  $Nu$  number determined and validated in relation to  $Ra$  number for the porous medium case for a heat flux of  $905 \text{ W/m}^2$  given by the expression (13).

A comparison of the findings was made using Table 3b showing clearly the positive effect of the utilisation of the aluminium net inside the pipes of the collector section. A future task would be to incorporate the metallic mesh parameter into the computer algorithm in order to enhance its performance. This application can be used as practical demonstration to students at a Lab exercise.

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