Journal of Engineering Science and Technology Review 6 (4) (2013) 1-8
Special Issue on Recent Advances in Nonlinear Circuits: Theory and Applications

JOURNAL OF
Engineering Science and Technology Review

Research Article

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Simulation and Experimental Realization of Multi-Scroll Chaotic Oscillators

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Received 30 July 2013; Revised 12 September 2013; Accepted 25 September 2013

Abstract

This article shows the simulation and experimental realization of a multi-scroll chaotic oscillator based on saturated nonlinear function (SNLF) series. First, the simulation is performed by MATLAB using the mathematical description of the oscillator. At this step, it is shown how to increase the number of scrolls by augmenting the SNLF. Second, the mathematical description is implemented with electronic devices and simulated by using the behavioral model of commercially available operational amplifiers. The circuit simulation of the multi-scroll chaotic oscillator is performed showing how to generate even and odd number of scrolls. Finally, the simulated circuit is realized experimentally and the results confirm good agreement with theoretical simulations.

Keywords: Chaos, multi-scroll chaotic oscillator, circuit simulation, operational amplifier.

1. Introduction

Chaotic oscillators have been implemented by a huge number of electronic devices [1]. Some of them have the capability of generating more than two scrolls, thus called multi-scroll chaotic oscillators. Basically, continuous time chaotic oscillators can be designed from third order dynamical systems, i.e. having three state variables that are associated to three first-order ordinary differential equations. One class of multi-scroll chaotic oscillators can be implemented using piecewise-linear (PWL) functions, which generally are augmented in linear segments to increase the number of scrolls [2]. In addition, as described and summarized in [1], PWL-function based chaotic oscillators have low design difficulty for electronics realizations. That way, those kind of multi-scroll chaotic oscillators are good candidates not only for experimental realization, but also for automating their design process from high-level simulation down to their electronic implementation, as shown in [2-4]. Another advantage of PWL-function based multi-scroll attractors is their versatility to design chaotic oscillators in more than one direction (1D), e.g. 2D and 3D as described in [3].

The majority of electronic realizations of chaotic oscillators use operational amplifiers (op-amps) [1-5], showing good chaotic behavior. However, that designs can be improved by optimizing the maximum Lyapunov exponent value, as shown in [6-7]. As a result, PWL-function based multi-scroll chaotic oscillators are good candidates for developing engineering applications, as the ones described in [5, 8-11].

In this article, we highlight the mathematical simulation, the circuit simulation, and the experimental realization of a multi-scroll chaotic oscillator consisting of saturated nonlinear function (SNLF) series that are approximated by PWL functions, and which are divided into two types of SNLFs: one for generating even number of scrolls; and the other one for generating odd number of scrolls.

Section 2 shows the mathematical description of the multi-scroll chaotic oscillator, and the model of the SNLF for generating even and odd number of scrolls. MATLAB simulations are performed to generate from 2 to 5 scrolls. Section 3 shows the electronic implementation of the SNLF and the whole chaotic oscillator, in order to simulate the circuit by using behavioral models of commercially available operational amplifiers. Section 4 shows experimental results for generating even and odd number of scrolls. Finally, the conclusions are listed in Section 5.

2. Multi-scroll Chaotic Oscillator

The mathematical description of the saturated nonlinear function (SNLF) series based multi-scroll chaotic oscillator can be modeled by [2, 11],

$$\begin{cases} \dot{x} = y \\ \dot{y} = z \\ \dot{z} = -ax - by - cz + df(x; k, h, p.q) \end{cases} \tag{1}$$

where,

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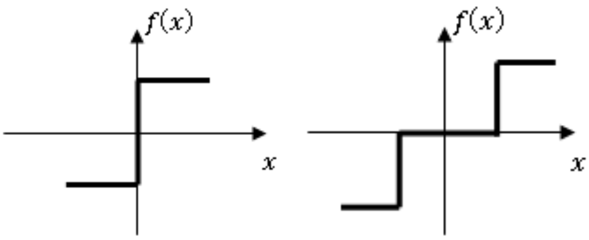
ISSN: 1791-2377

$f(x;\, k,\, h,\, p,\, q)$: Saturated nonlinear function (SNLF) series description
x, y, z: State variables
a, b, c, d: Real and positive constants satisfying: $0 < a, b, c, d < 1$.

Equivalently, Eq. (1) can be described in matrix form like

$$\begin{bmatrix}\dot{x}\\ \dot{y}\\ \dot{z}\end{bmatrix}=\begin{bmatrix}0 & 1 & 0\\ 0 & 0 & 1\\ -a & -b & -c\end{bmatrix}\begin{bmatrix}x\\ y\\ z\end{bmatrix}+\begin{bmatrix}0\\ 0\\ d\end{bmatrix}f(x;k,h,p,q) \qquad (2)$$

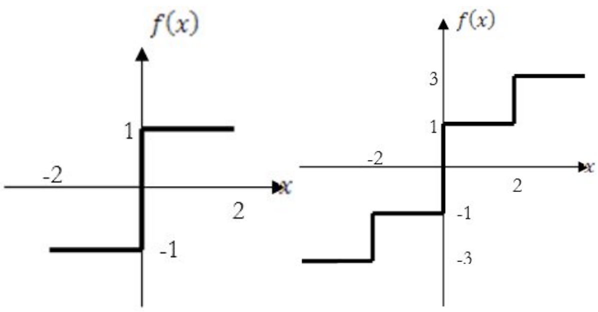
The SNLF is divided into two kinds of functions to generate even and odd number of scrolls, as already shown in [2]. In this section, we just show the abstract and graphical description of the SNLF for generating two and four scrolls for the even case, and three and five scrolls for the odd case. For instance, Fig. 1a shows the seed of the SNLF for generating even number of scrolls, and Fig. 1b shows the seed for generating odd number of scrolls. In each case the number of scrolls to be generated is the number of saturated levels. Basically, in each case the SNLF is augmented in saturated levels, which are systematically shifted as shown in Fig. 2 and Fig. 3.



(a) (b)

Fig. 1. SNLF description for generating (a) two, and (b) three scrolls.

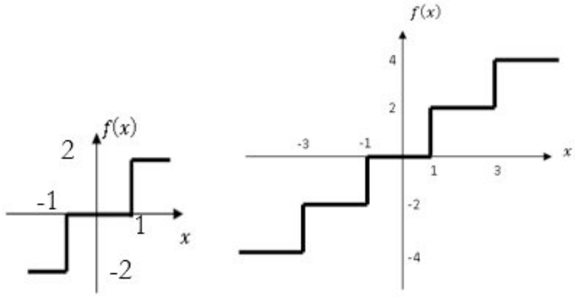
The simulation of Eq. (1) requires setting the number of scrolls to be generated, it means the designer should set the values of the four coefficients *a, b, c, d,* the amplitude of the saturated levels and the break points (shifts) and slopes of the SNLF. For example, Fig. 2 shows the values of the amplitudes and the break points of the SNLFs to generate 2 and 4 scrolls.



(a) (b)

Fig. 2. SNLF to generate (a) two, and (b) four scrolls.

Similarly, the values of the amplitudes and the break points of the SNLFs to generate 3 and 5 scrolls, respectively, are shown in Fig. 3. As one can infer, Fig. 2 or Fig. 3 can be systematically augmented to increase the number of even or odd number of scrolls, respectively.



(a) (b)

Fig. 3. SNLF to generate (a) three, and (b) five scrolls.

Assuming the slope of the SNLF to be equal to 10,000, as provided by commercially available operational amplifiers, and by setting the coefficient values of $a = b = c = d = 0.7$, as usually done [2, 11], then Fig. 4 shows the MATLAB simulations of the SNLF, the state variable x vs. time, and the phase state portrait for generating two scrolls. In the same manner, using Fig. 2 and Fig. 3 for the SNLF with slope 10,000, and $a = b = c = d = 0.7$, Figs. 5 to 7 show the MATLAB simulations for generating 3 to 5 scrolls.

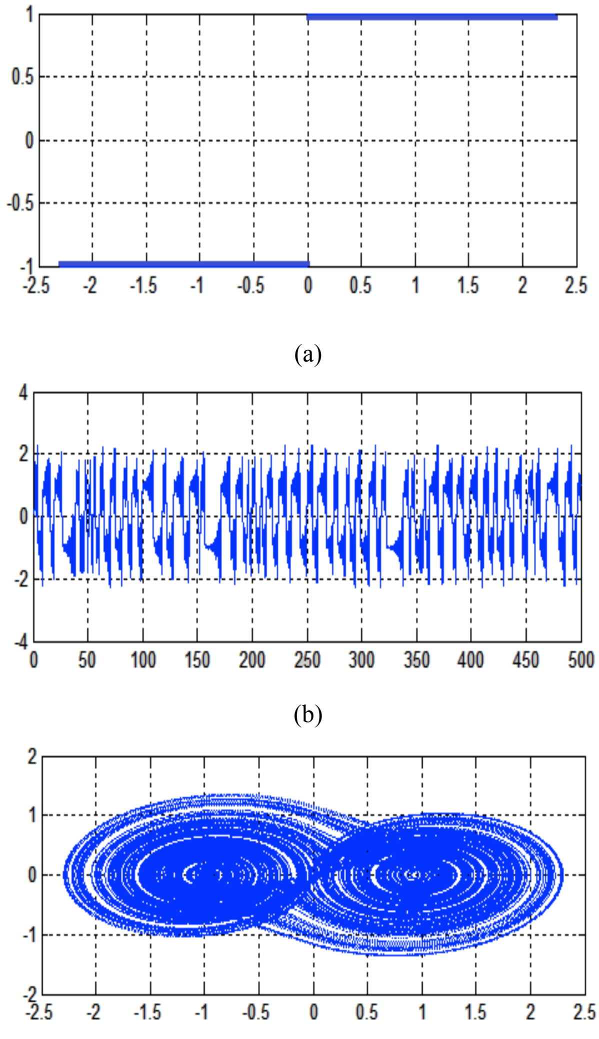


Fig. 4. (a) Simulation of the SNLF shown in Fig. 2a, (b) state variable x from (1), and (c) x vs. y phase state portrait for generating two scrolls.

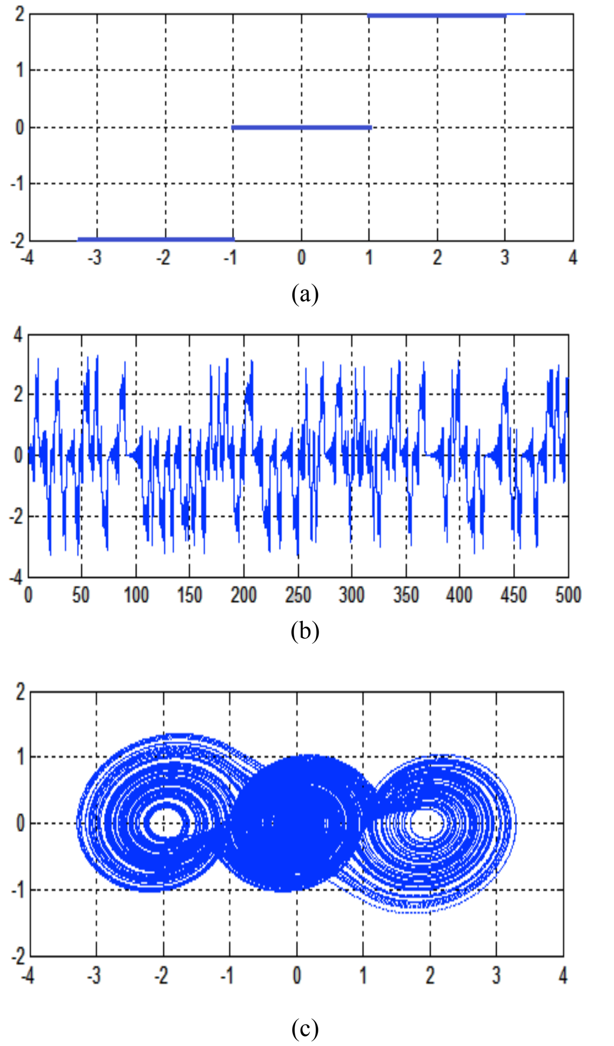
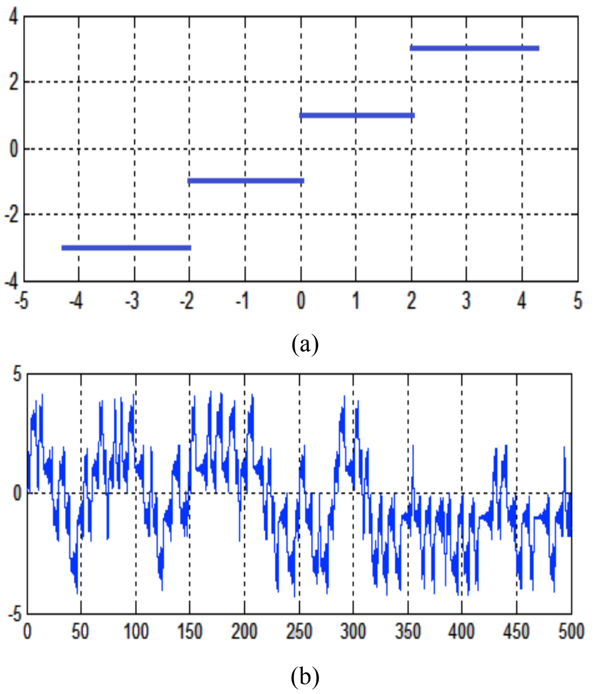


Fig. 5. (a) SNLF shown in Fig. 3a, (b) state variable *x*, and (c) *x* vs. *y* for 3 scrolls.



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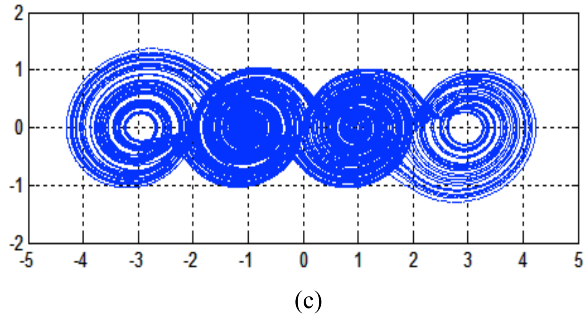


Fig. 6. (a) SNLF shown in Fig. 2b, (b) state variable *x*, and (c) *x* vs. *y* for 4 scrolls.

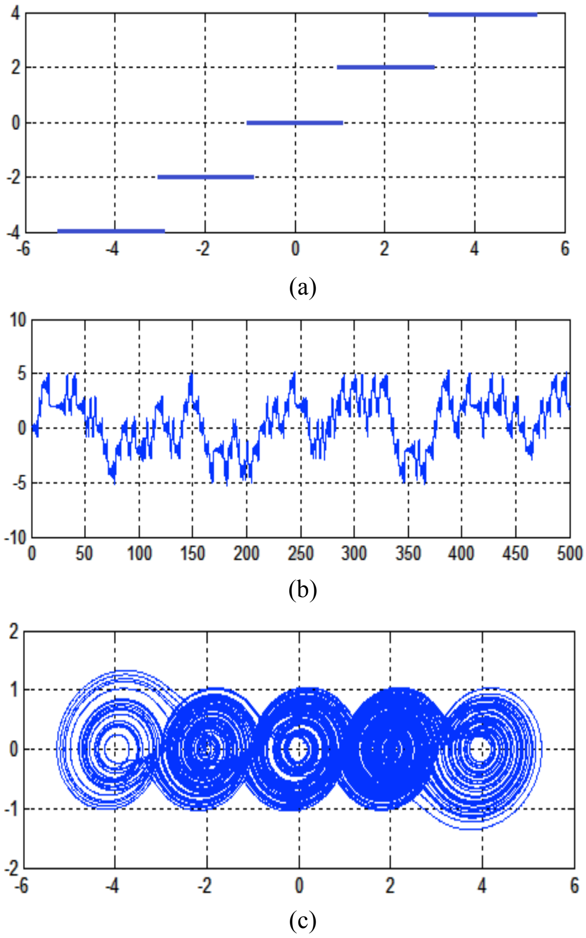


Fig. 7. (a) SNLF shown in Fig. 3b, (b) state variable *x*, and (c) *x* vs. *y* for 5 scrolls.

3. Circuit Implementation and Simulation

Using operational amplifiers (op-amps), the implementation of the SNLF in the voltage-current (v-i) plane, requires n-1 op-amps to generate n scrolls. For example, to implement the SNLF shown in Fig. 2b, three op-amps are required, each one is saturated in open loop and shifted in voltage as required [2, 11]. That SNLF requires 3 op-amps because it has 4 saturated levels at -3, -1, 1 and 3, and two shifted voltages (break-points) at -2 and 2. The op-amp-based implementation is shown in Fig. 8, where Ei1 and Ei3 perform the shifted voltages -2 and 2, while Ei2 is set to zero [2]. The saturated levels of each op-amp are then converted to current through Ri1, Ri2 and Ri3. The currents are added in one node and then converted to voltage using a transresistance amplifier with gain established by Rf. The resistances Ri1, Ri2, Ri3 and Rf then take values to provide the amplitudes 3, 1, -1, and -3.

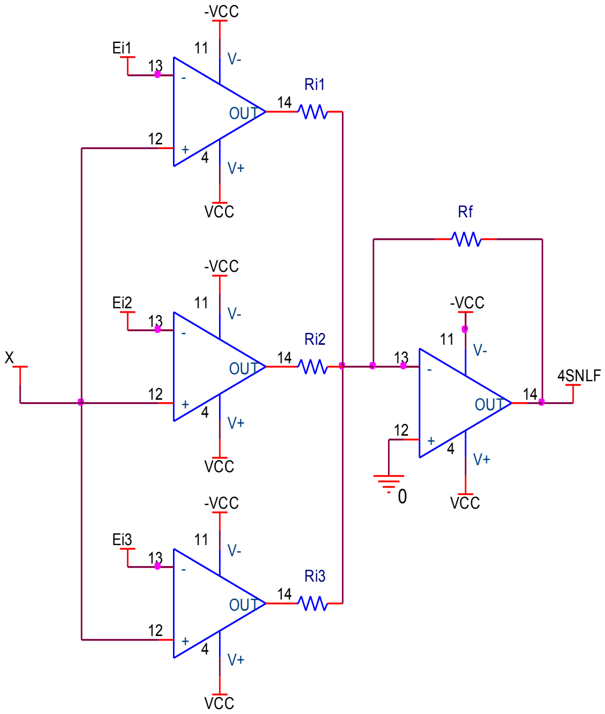


Fig. 8. Implementing the SNLF from Fig. 2b to generate 4 scrolls.

Figure 9 shows the implementation of the system described by Eq. (1) using op-amps. As one sees, three integrators are used; each one in inverter topology thus requiring another inverter to generate the positive sign of the state variables *x, y, z*, and which are implemented by the op-amps with external resistors Rp and Rq. The third equation (row) in system's Eqs. (1) requires an adder that is implemented by the op-amp with external resistors Rx1-Rx4 and Ry. Then each coefficient (*a, b, c, d*) is implemented with the op-amps having external resistors Ri and Rf. Finally, the op-amp with external resistors Rii and Rff is used to invert one sign.

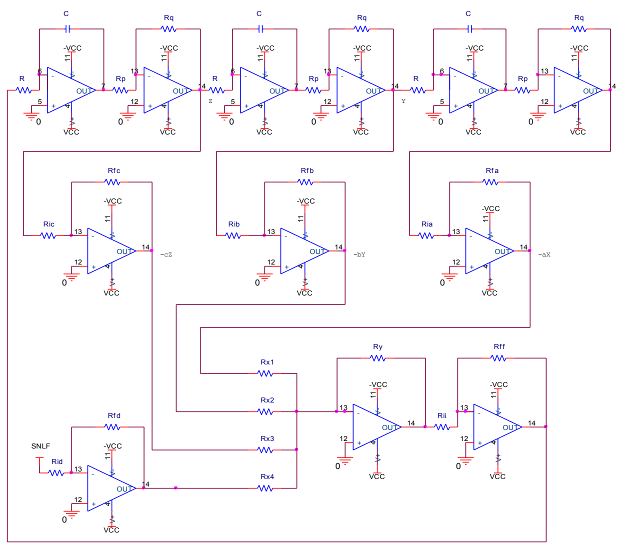


Fig. 9. Implementation of the system's Eqs. (1) using op-amps.

Tables 1 and 2 summarize the circuit element values and the values of the independent sources used to generate even (2 and 4) and odd (3 and 5) number of scrolls.

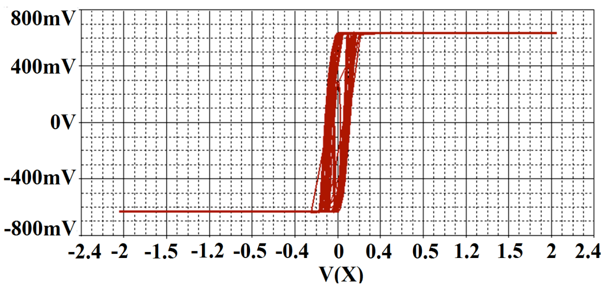
Table 1. Circuit element values to generate 2 to 5 scrolls.

Scrolls	Element Values
2	Rf=5.5 KΩ, Ri1= 100 KΩ, VCC= 18 V, -VCC= 18 V, Ei1= 0 V
3	Rf=5.5 KΩ, Ri1, Ri2= 100 KΩ, VCC= 18 V, -VCC= 18 V, Ei1= -1 V, Ei2= 1 V
4	Rf=5.5 KΩ, Ri1, Ri2, Ri3 = 100 KΩ, VCC= 18 V, -VCC= 18 V, Ei1= 0 V, Ei2= -2 V, Ei3= 2 V
5	Rf=5.5 KΩ, Ri1, Ri2, Ri3, Ri4 = 100 KΩ, VCC= 18 V, -VCC= 18 V, Ei1= -1 V, Ei2= 1 V, Ei3= -3 V, Ei4= 3 V

Table 2. Circuit element values in Fig. 9.

Element	Values
C	100 Pf
R	1 MΩ
Rp, Rq	1 KΩ
Rfa, Rfb, Rfc, Rfd	7 KΩ
Rx1, Rx2, Rx3, Rx4, Ry	100 KΩ
Ria, Rib, Ric, Rid, Rii, Rff	10 KΩ

Figures 10-13 show simulation results of Fig. 9 for generating from 2 to 5 scrolls using Multisim [12]. The values of the resistors and capacitors are given in Tables 1 and 2 to accomplish Eq. (1) and Figs. 2 and 3, as discussed above.



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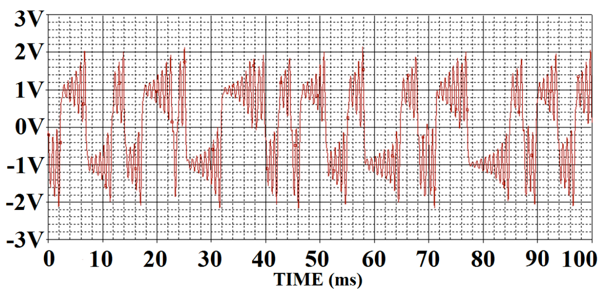

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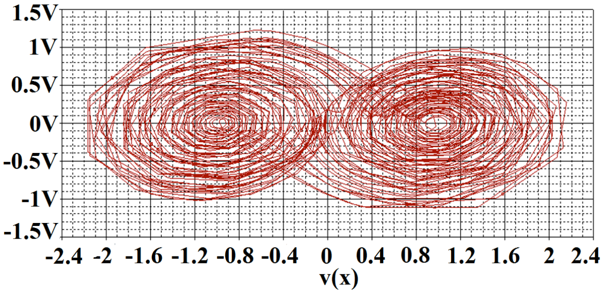

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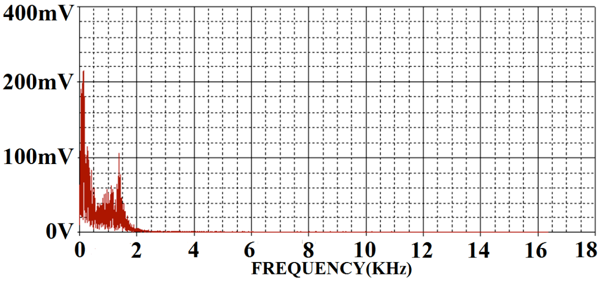
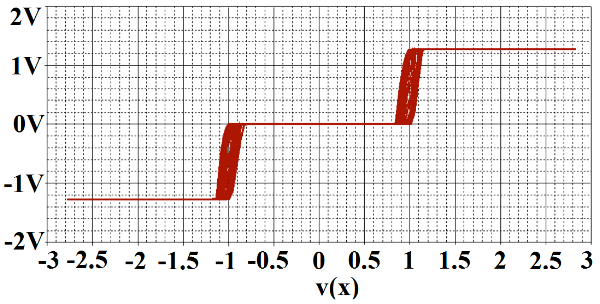


(d)

Fig. 10. (a) SNLF, (b) state variable x vs. time, (c) phase space portrait between state variables x-y for generating 2-scrolls, and (d) Fourier spectrum of state variable x.



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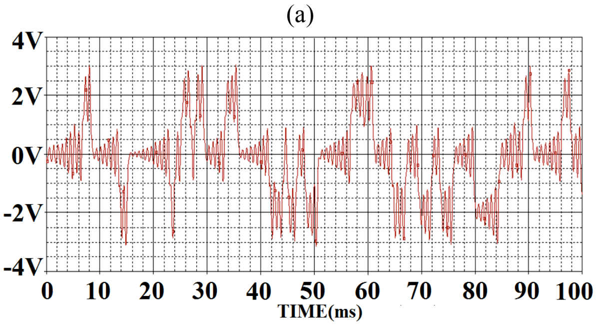

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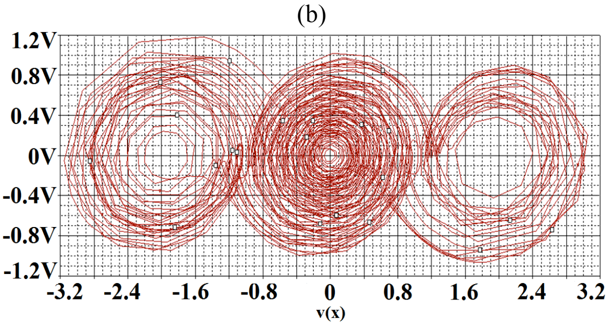

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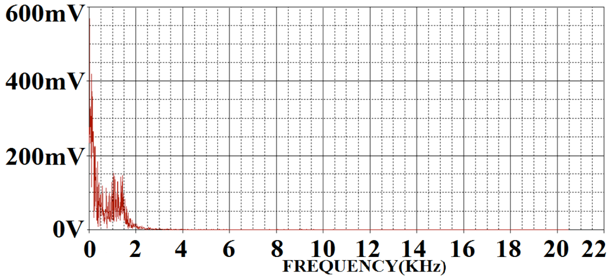
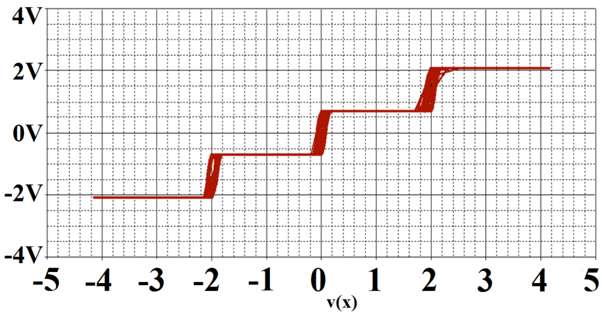


(d)

Fig. 11. (a) SNLF, (b) state variable *x* vs. time, (c) phase space portrait for generating 3-scrolls, and (d) Fourier spectrum of *x*.



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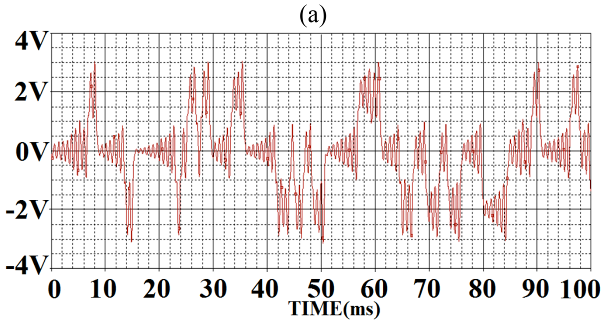

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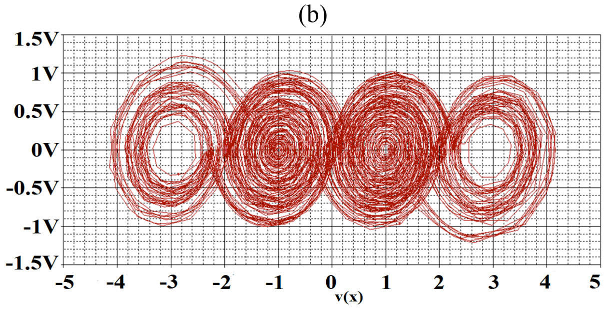

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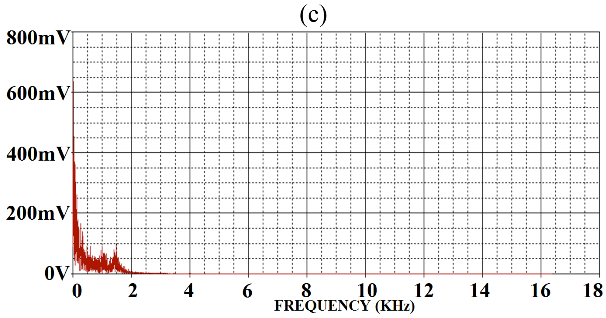


(d)

Fig. 12. (a) SNLF, (b) state variable x vs. time, (c) phase space portrait for generating 4-scrolls, and (d) Fourier spectrum of x.

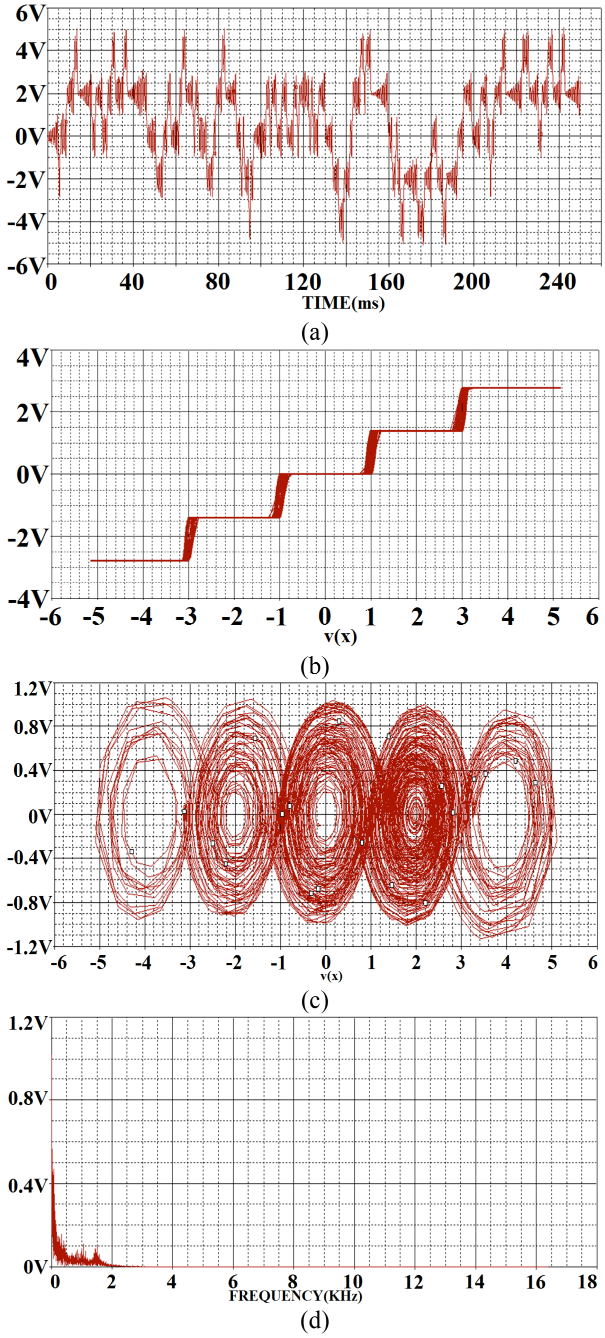


Fig. 13. (a) SNLF, (b) state variable *x* vs. time, (c) phase space portrait for generating 5-scrolls, and (d) Fourier spectrum of *x*.

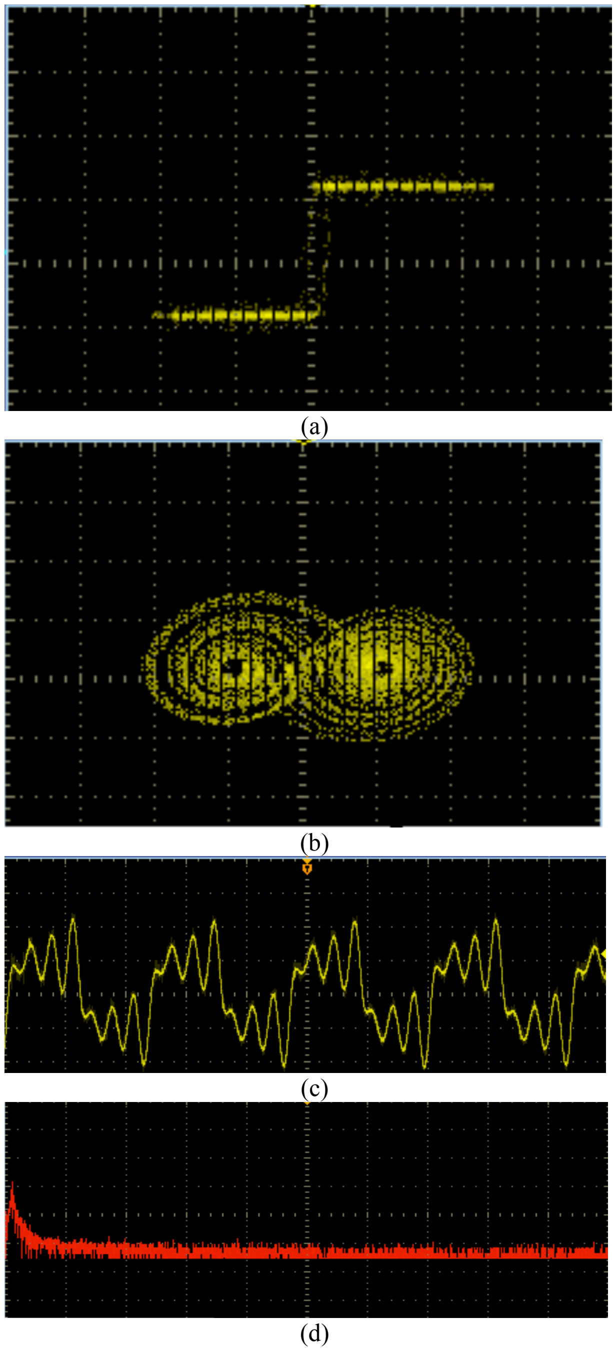
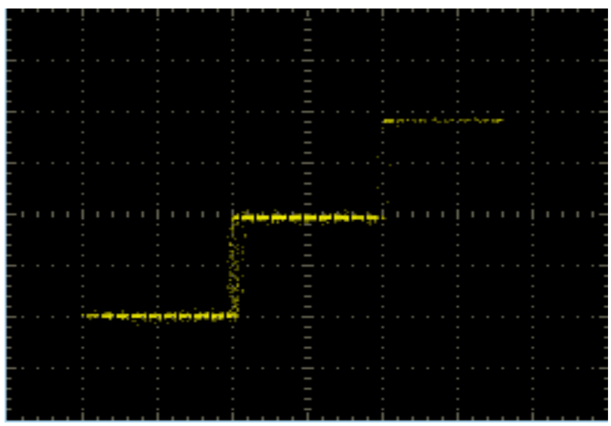


Fig. 14. Scale: V/div = 500 mV, t = 2 ms, (a) SNLF, (b) phase space portrait *x-y*, (c) state variable *x* vs. time, and (d) Fourier spectrum of *x* (20dB-12.5kHz, Nyquist 130.2kHz).

4. Experimental Results

The circuits described in the previous section were realized under laboratory conditions. For tuning the values of the coefficients *a, b, c, d*, linear precision potentiometers were used. That way, the results for generating from 2 to 5 scrolls are shown in the experimental results depicted in Figs. 14 to 17, respectively.



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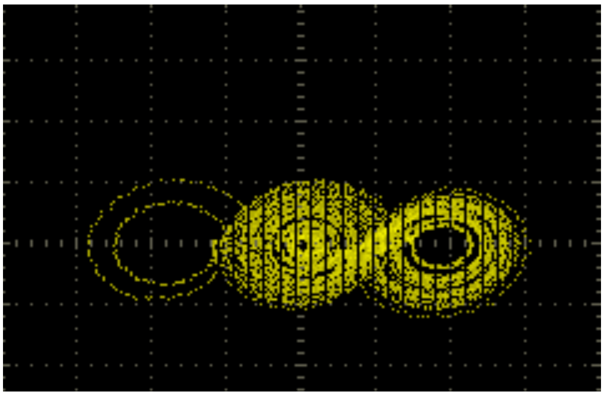

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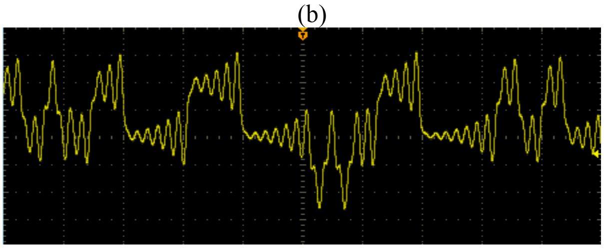

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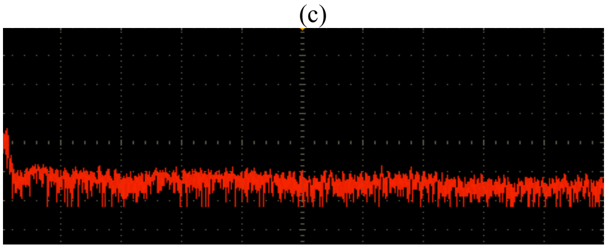
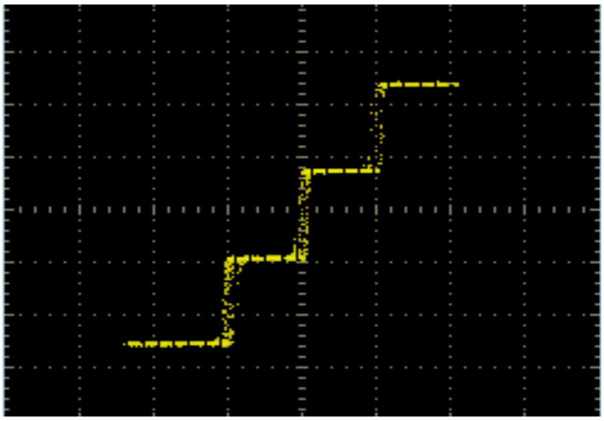


(d)

Fig. 15. Scale: V/div = 100 mV, t = 4 ms, (a) SNLF, (b) phase space portrait *x-y*, (c) state variable *x* vs. time, and (d) Fourier spectrum of *x* (20dB-12.5kHz, Nyquist 130.2kHz).



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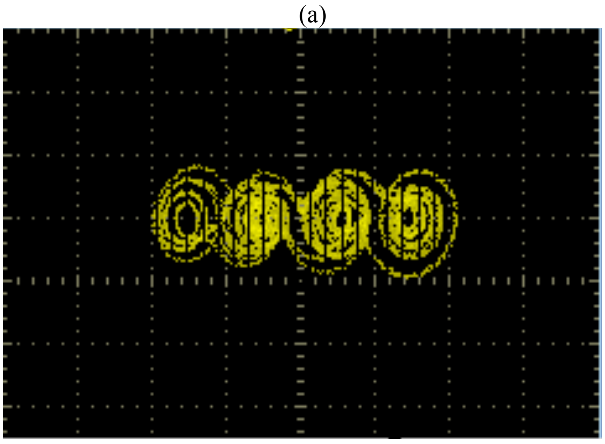

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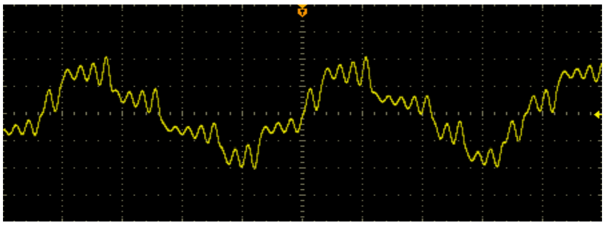

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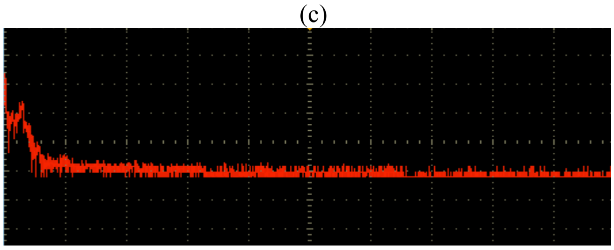

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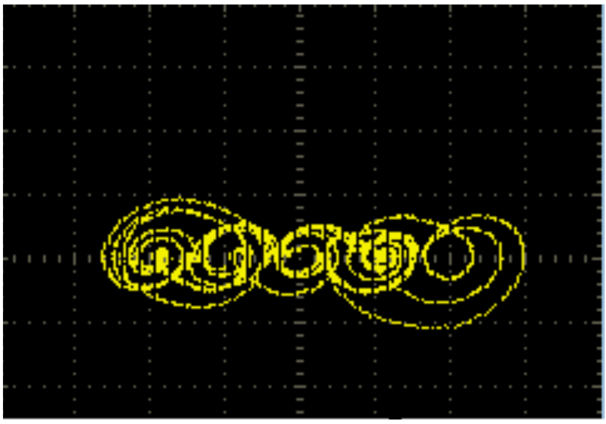
Fig. 16. Scale: V/div = 2 V, t = 4 ms, (a) SNLF, (b) phase space portrait *x-y*, (c) state variable *x* vs. time, and (d) Fourier spectrum of x (20dB-5kHz, Nyquist 65.1kHz).



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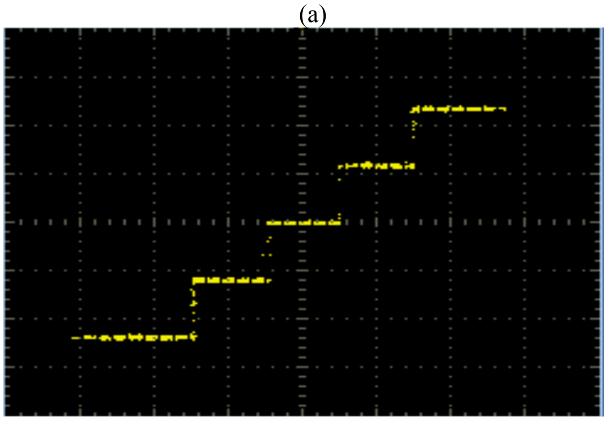

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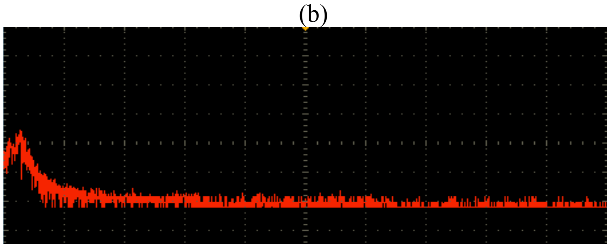

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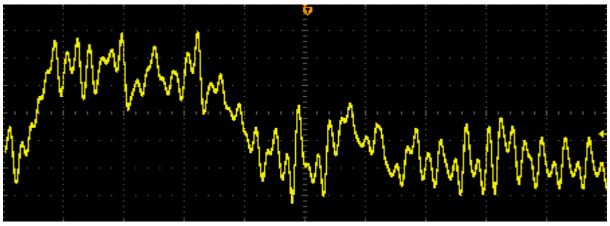

(d)

Fig. 17. Scale: V/div = 200 mV, t = 40 ms, (a) SNLF, (b) phase space portrait *x-y*, (c) state variable *x* vs. time, and (d) Fourier spectrum of *x* (20dB-500Hz, Nyquist 6.51kHz).

The experimental results provided herein could be used in applications like the one introduced in [5]. Besides, the coefficients a, b, c, d, can be varied to generate more chaos that can be quantified by evaluating the Lyapunov exponents, as already shown in [6]. That way, one have different combinations of coefficient values, but also one can vary the amplitude, break points and slopes of the SNLFs.

5. Conclusion

Chaotic oscillators can be simulated from the abstract mathematical description and then one can refine the circuit implementation from circuit simulation down to experimental realization, as showed herein. In particular, the multi-scroll chaotic oscillator based on SNLF series can be implemented using op-amps, where the generation of even and odd number of scrolls can be done by increasing the number of segments of the SNLF, as shown in Section 3. Finally, the simulation results from the mathematical description and circuit simulation for generating from 2 to 5 scrolls, showed good agreement with experimental results, thus concluding on the appropriateness of performing abstract simulations, and ending with the experimental realization of chaos generators.

Acknowledgment

This work was partially supported by Delfin Program at Mexico and CONACyT-Mexico under project 131839-Y.

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