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Research Article

Design and Numerical Simulation of Unidirectional Chaotic Synchronization and its Application in Secure Communication System

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Abstract

Chaotic systems are characterized by sensitive dependence on initial conditions, similar to random behavior, and continuous broad-band power spectrum. Chaos is a good potential to be used in secure communications system. In this paper, in order to show some interesting phenomena of three-order Jerk circuit with modulus nonlinearity, the chaotic behavior as a function of a variable control parameter, has been studied. The initial study in this paper is to analyze the phase portraits, the Poincaré maps, the bifurcation diagrams, while the analysis of the synchronization in the case of unidirectional coupling between two identical generated chaotic systems, has been presented. Moreover, some appropriate comparisons are made to contrast some of the existing results. Finally, the effectiveness of the unidirectional coupling scheme between two identical Jerk circuits in a secure communication system is presented in details. Integration of theoretical physics, the numerical simulation by using MATLAB 2010, as well as the implementation of circuit simulations by using MultiSIM 10.0 has been performed in this study.

Keywords: Jerk circuit, synchronization, Poincaré map, bifurcation diagram, unidirectional coupling, secure communication system.

1. Introduction

Chaos is used to describe the behavior of certain dynamical nonlinear systems, i.e., systems which state variables evolve with time, exhibiting complex dynamics that are highly sensitive on initial conditions. As a result of this sensitivity, which manifests itself as an exponential growth of perturbations in the initial conditions, the behavior of chaotic systems appears to be almost random [1]. Chaotic behavior has been found in physics [2], chemistry [3], biology [4], robotics [5], bits generators [6], psychology [7], ecology [8-9], cryptography [10] and economy [11].

During the last decade, synchronization of chaotic systems has been explored very intensively by many researchers by using electronic circuits, such as Rössler circuit [12], Duffing circuit [13], Chua circuit [14], Double Bell circuit [15,16], FitzHugh-Nagumo Neuronal System [17,18] and Hindmarsh-Rose Neuron Model [19]. For the latter, the main motivation has been potential practical applications in communication systems [20, 21].

Pecora and Carroll first demonstrated how chaotic systems could be synchronized, using an electronic circuit coupled unidirectional to a subsystem made up of components of the parent system [22].

This innovation provided a new perspective in chaotic dynamics and inspired many studies on the synchronization of chaotic systems. Cuomo and Oppenheim further expanded the area by demonstrating how synchronized chaotic systems could be used in a scheme for secure communication [23].

Synchronization between chaotic signals has received considerable attention and led to various communication applications. Many researchers demonstrated, using simulation, that chaos can be synchronized and applied to secure communication schemes, such us, in secure fiber-optical communication scheme using chaos [24], in secure communication based on chaotic cipher [25] and in secure communication with chaotic lasers [26].

The plan of the paper is as follows. In section 2, the details of the proposed autonomous Jerk circuit's simulation using MATLAB 2010 and MultiSIM 10.0, are presented. In Section 3, the unidirectional coupling method is applied in order to synchronize two identical autonomous Jerk circuits. The chaotic masking communication scheme by using the above mentioned synchronization technique is presented in Section 4. Finally, in Section 5, the concluding remarks are given.

2. Jerk Circuit

Sprott found the functional form of three-dimensional dynamical systems which exhibit chaos. Jerk equation has a simple nonlinear function, which can be implemented with

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an autonomous electronic circuit. In this work, the Jerk circuit, which was firstly presented by Sprott in 2000 [27], is used. This is a three-dimensional autonomous nonlinear system that is described by the following system of ordinary differential equations:

$$\left.\begin{aligned} \dot{x} &= y \\ \dot{y} &= z \\ \dot{z} &= -az - by + |x| - 1 \end{aligned}\right\} \tag{1}$$

This equation has only one nonlinear term in the form of absolute value of the variable x. The parameters and initial conditions of the Jerk system (1) are chosen as: *(a, b) = (0.6, 1)* and (x_0, y_0, z_0) = (0, 0, 0), so that the system shows the expected chaotic behavior. The Jerk system has two equilibrium points (1, 0, 0) and (−1, 0, 0).

For equilibrium points (1, 0, 0), the Jacobian becomes

$$J_1 = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & -b & -a \end{pmatrix} \tag{2}$$

The eigenvalues are obtained by solving the characteristic equation, $\det[\lambda I - J_1] = 0$ which is:

$$\lambda^3 + 0.6\lambda^2 + \lambda + 1 = 0 \tag{3}$$

Yielding eigenvalues of λ_1 = -0.835551, λ_2 = 0.117776 - 1.08763i, λ_3 = 0.117776 + 1.08763i, for a = 0.6, b = 1. For equilibrium points (-1, 0, 0), the Jacobian becomes:

$$J_2 = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & -b & -a \end{pmatrix} \tag{4}$$

The eigenvalues are obtained by solving the characteristic equation, $\det[\lambda I - J_1] = 0$ which is:

$$\lambda^3 + 0.6\lambda^2 + \lambda - 1 = 0 \tag{5}$$

Yielding eigenvalues of λ_1 = 0.588458, λ_2 = -0.594229 - 1.16028i, λ_3 = -0.594229 + 1.16028i, for a = 0.6, b = 1.

The above eigenvalues show that the system has an unstable spiral behavior. In this case, the phenomenon of chaos is presented.

2.1 Numerical Simulations

In this section, numerical simulations are carried out by using MATLAB 2010. The fourth-order Runge-Kutta method is used to solve the system of differential equations (1). Figs.1(a)-(c) show the projections of the phase space orbit on to the x–y plane, the y–z plane and the x–z plane, respectively. As it is shown, for the chosen set of parameters and initial conditions, the Jerk system presents chaotic attractors of Rössler type. Also, it is known from the nonlinear theory, that the spectrum of Lyapunov exponents provides additional useful information about system's behavior. In a three dimensional system, like this, there has been three Lyapunov exponents (LE_1, LE_2, LE_3). In more details, for a 3D continuous dissipative system the values of the Lyapunov exponents are useful for distinguishing among the various types of orbits. So, the possible spectra of attractors, of this class of dynamical systems, can be classified in four groups, based on Lyapunov exponents [28-31].

- (LE_1, LE_2, LE_3) → (–, –, –): a fixed point
- (LE_1, LE_2, LE_3) → (0, –, –): a limit point
- (LE_1, LE_2, LE_3) → (0, 0, –): a two-torus
- (LE_1, LE_2, LE_3) → (+, 0, –): a strange attractor (Fig.1).

So, from the diagram of Lyapunov exponents of Jerk's system of Fig.2, the expected chaotic behavior, from the same set of parameters and initial conditions can be concluded.

Bifurcation theory was originally developed by Poincaré. It is used to indicate the qualitative change in system's behavior, in terms of the number and the type of solutions, under the variation of one or more parameters on which the system depends [32, 33]. To observe the system dynamics under all the above possible bifurcations, a bifurcation diagram may be constructed, which shows the variation of one of the state variables with one of the control parameters. A MATLAB program was written to obtain the bifurcation diagrams for Jerk circuit of Figs.3(a)-(c). So, in this diagram a possible bifurcation diagram for system (1), in the range of $0.55 \le a \le 1$, is shown. For the chosen value of $0.55 \le a \le 0.6$ the system displays the expected chaotic behavior. For $0.6 < a \le 0.76$, a period-2 behavior of the system is observed, and finally for $a > 0.76$ a period-1 behavior system is present.

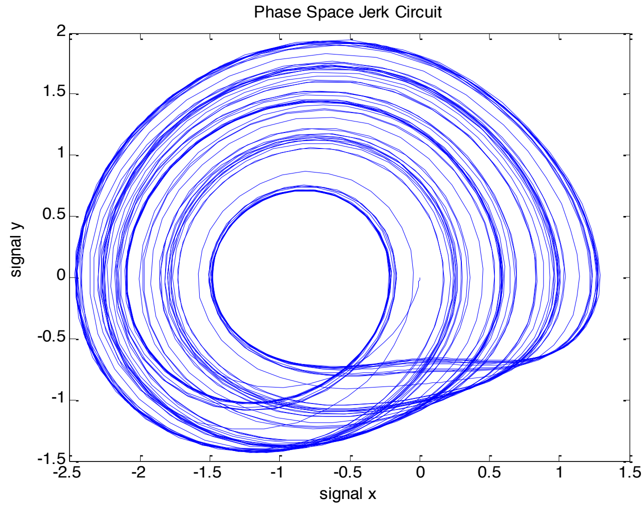
Another useful tool for analyzing the dynamical characteristics of a nonlinear system is the Poincaré map. In the chaotic state the phase portrait is very dense, in the sense, that the trajectories of the motion are very close to each other. It can be only indicative of the minima and maxima of the motion. Any other characterization of the motion is difficult to be interpreted. So, one way to capture the qualitative features of the strange attractor is to obtain the Poincaré map [32,34]. Figs.4(a)-(c) shows the Poincaré section map by using MATLAB, for a = 0.6, b = 1.

2.2 Analog Circuit Simulation Using MultiSIM

The designed circuitry, which realizes system (1) is shown in Fig.5. The circuit has also a basin of attraction outside of which the dynamics are unbounded, which manifests the saturation of the op-amps. If the op-amps are saturated, it is necessary to restart the circuit. The relationship among the resistors R, R_A used in the circuit and the parameter 'a' mentioned in (1) is given below. Here R and R_A are measured in kΩ.

$$R_A = \frac{R}{a} \tag{6}$$

where $R_A = R_2$ = 2 kΩ and R = 1 kΩ. The occurrence of the chaotic attractor can be clearly seen from Figs.6(a)–(c). By comparing Figs.1(a)-(c) with Figs.6(a)-(c) a good qualitative agreement between the numerical integration of (1) by using MATLAB 2010, and the circuit's simulation by using MultiSIM 10.0, can be concluded.



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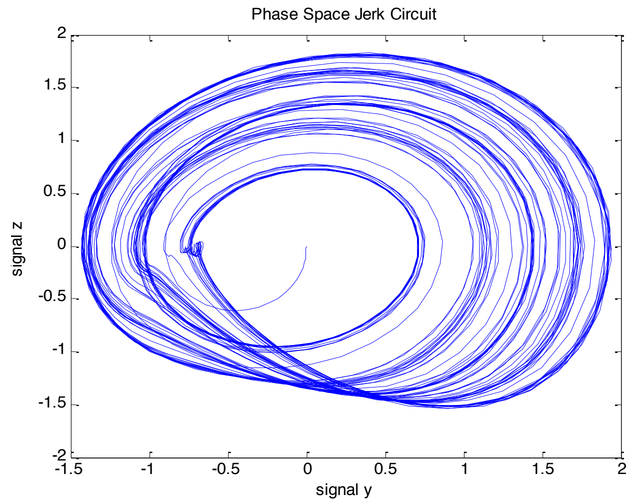

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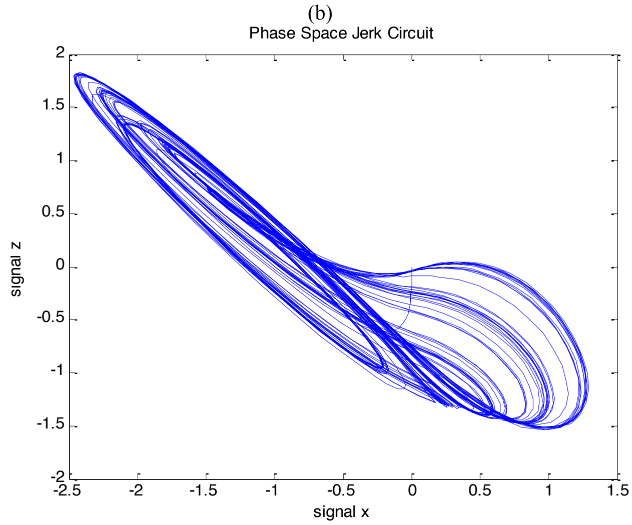


(c)

Fig. 1. Numerical simulation results using MATLAB 2010, for a = 0.6, b = 1, in (a) *x-y* plane, (b) *y-z* plane, (c) *x-z* plane.

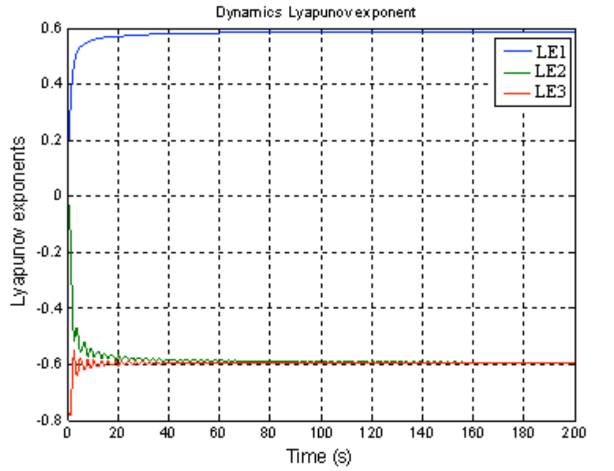
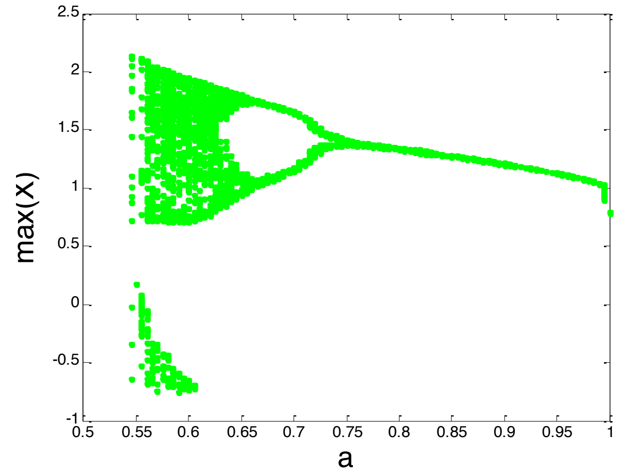
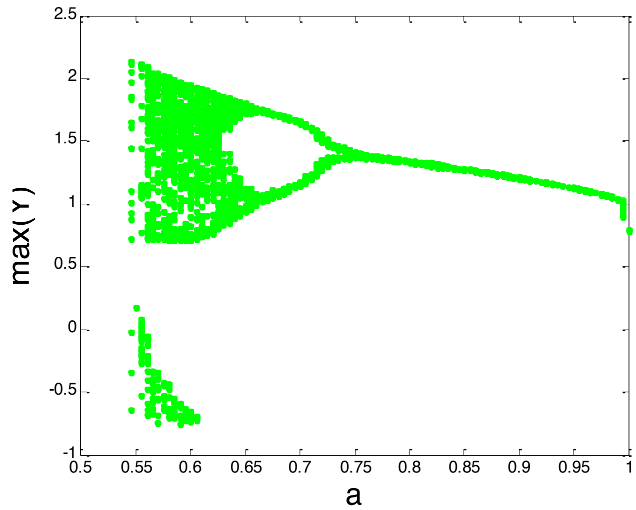


Fig. 2. The dynamics of Lyapunov exponents of the Jerk system, for $a = 0.6$, $b = 1$.



(a)



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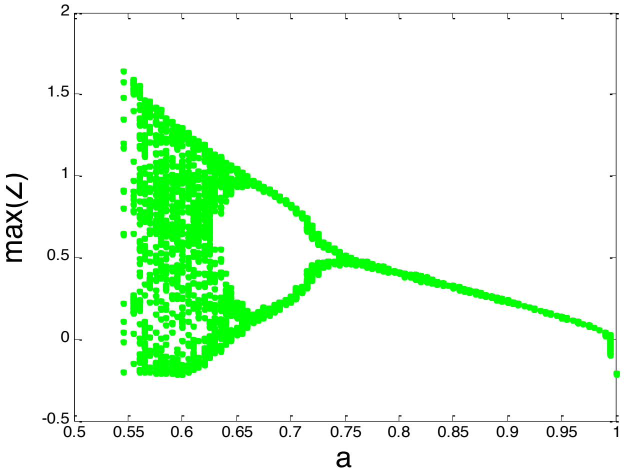
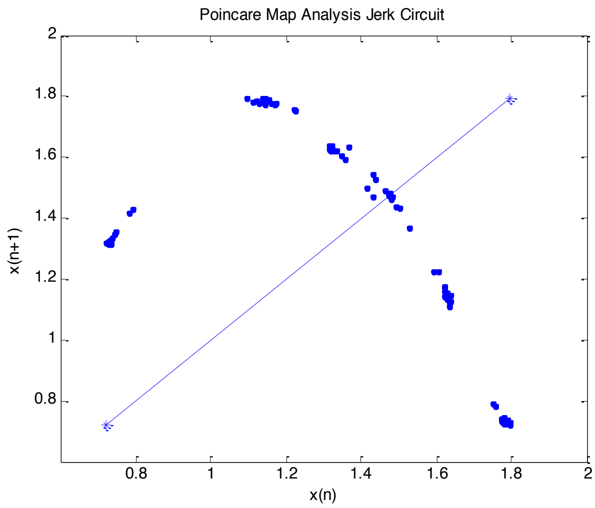
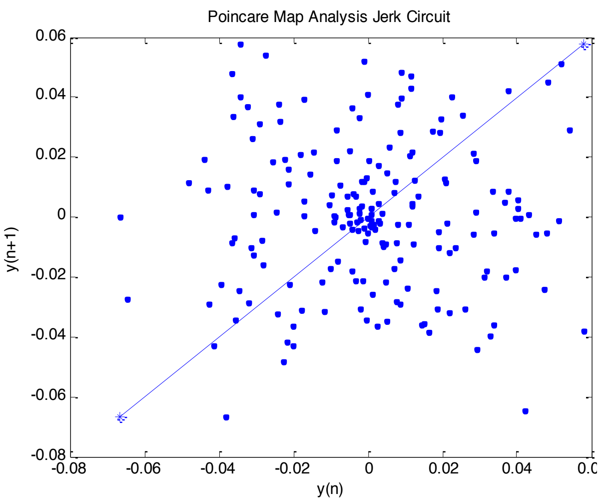


(c)

Fig. 3. (a) Bifurcation diagram of x vs. the control parameter $a \in [0.5-1]$, (b) Bifurcation diagram of y vs. the control parameter $a \in [0.5-1]$ and (c) Bifurcation diagram of z vs. the control parameter $a \in [0.5-1]$, for the specific values (a = 0.6, b = 1), with MATLAB 2010.



(a)



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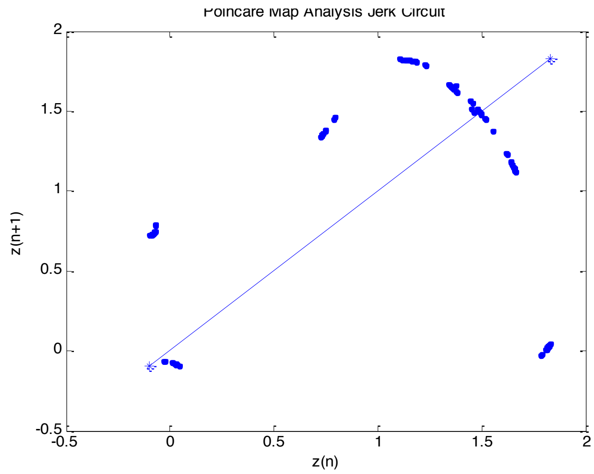


(c)

Fig. 4. A gallery of Poincare maps for system (1), for a = 0.6, b = 1: (a) the plot given the maxima of $x(n + 1)$ versus those of $x(n)$; (b) the plot given the maxima of $y(n + 1)$ versus those of $y(n)$; (c) the plot given the maxima of $z(n + 1)$ versus those of $z(n)$, with MATLAB 2010.

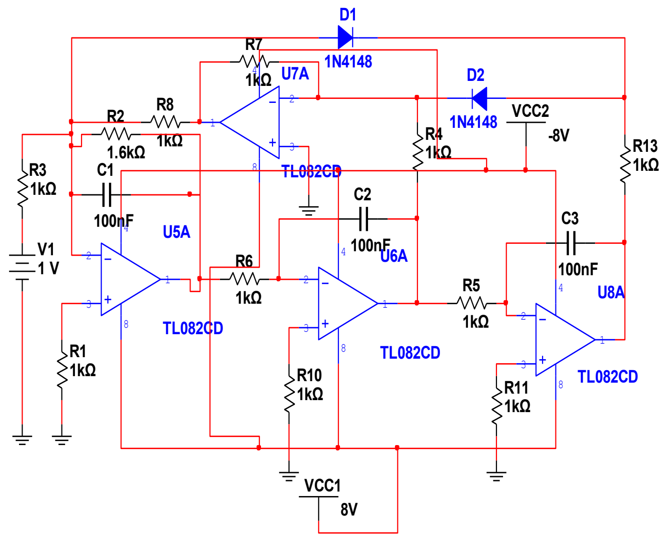


Fig. 5. Schematic of the proposed Jerk circuit using MultiSIM 10.0.

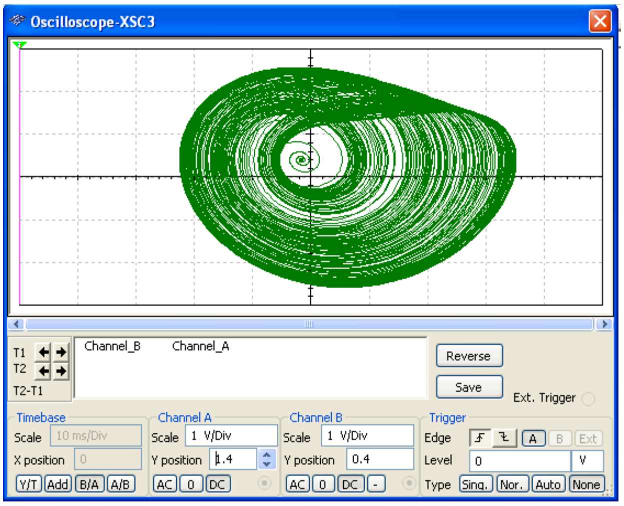
3. Unidirectional Chaotic Synchronization of Coupled Jerk Circuits

The general scheme for synchronization here is to take a driving system, create a subsystem and drive this and a duplicate of this subsystem, called a response or slave system, with signals from the drive system. In unidirectional synchronization, the evolution of the first system (the master) is unaltered by the coupling, while the second system (the slave) is then constrained to copy the dynamics of the first [35]. The following master-slave (unidirectional coupling) configuration is described below:

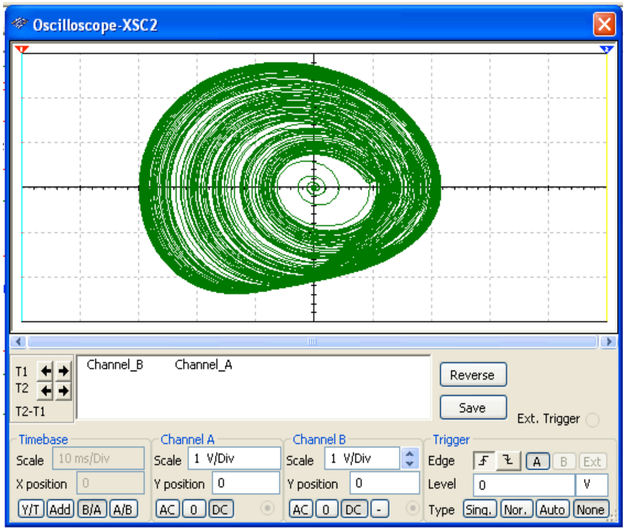
$$\left.\begin{aligned} &\textit{Master} \\ &\dot{x}_2 = y_2 \\ &\dot{y}_2 = z_2 \\ &\dot{z}_2 = -az_2 - by_2 + |x_2| - 1 \end{aligned}\right\} \tag{7}$$

$$\left.\begin{aligned} &\textit{Slave} \\ &\dot{x}_1 = y_1 \\ &\dot{y}_1 = z_1 + \xi(y_2 - y_1) \\ &\dot{z}_1 = -az_1 - by_1 + |x_1| - 1 \end{aligned}\right\} \tag{8}$$

where, ξ is the coupling factor. Numerical simulations are used to describe the dynamics of the phenomenon of unidirectional synchronization of Jerk circuit's systems (7-8) with fourth-order Runge-Kutta method.



(a)



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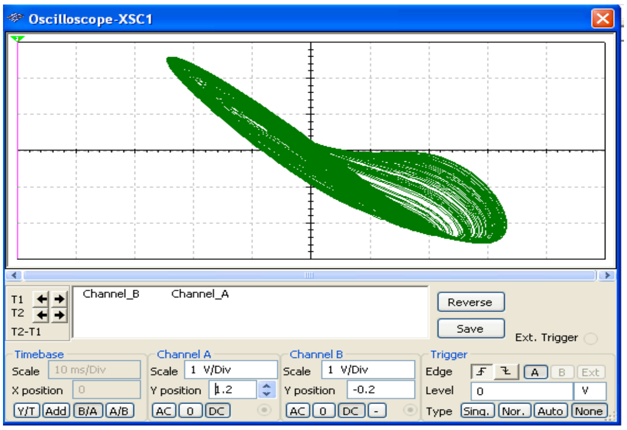


(c)

Fig. 6. Various projections of the chaotic attractor using MultiSIM 10.0, for a = 0.6, b = 1: (a) *x-y* plane (b) *y-z* plane (c) *x-z* plane.

In Fig.7 the bifurcation diagram of $(x_2 - x_1)$ versus ξ is shown. From this diagram a classical transition from the full desynchronization, for low values of coupling factor $(0 < \xi \leq 0.194)$, to full synchronization for higher values of coupling factor $(\xi > 0.194)$, is confirmed. Fig.8(a) shows the asynchronous phenomenon during coupling parameters $\xi = 0.184$, while Fig.8(b) shows the phenomenon of chaos synchronization when the coupling parameter $\xi = 0.250$ by using MATLAB 2010.

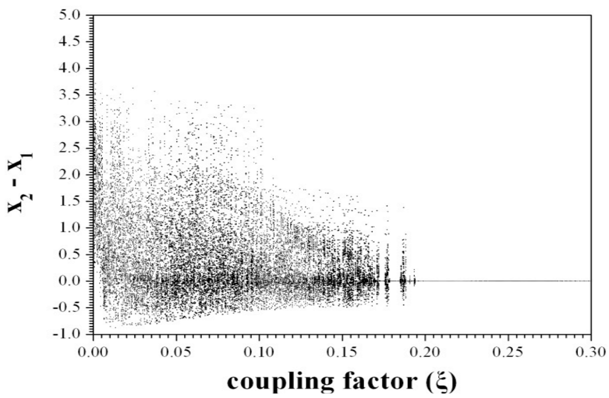


Fig. 7. Bifurcation diagram of $(x_2 - x_1)$ versus ξ, in the case of unidirectional coupling.

4. Application to Secure Communication Systems

To study the effectiveness of signal masking approach in the Jerk system, we first set the information-bearing signal $m_s(t)$ in the form of sinusoidal wave:

$$m_s(t) = A \sin(2\pi f)t \tag{9}$$

where A and f are the amplitude and the frequency of the sinusoidal wave signal respectively.

The sum of the signal $m_s(t)$ and the chaotic signal $m_{\text{Jerk\_circuit}}(t)$, produced by the Jerk circuit, is the new encryption signal $m_{\text{encryption}}$, which is given by Eq.(10).

$$m_{encryption}(t) = m_s(t) + m_{\text{Jerk\_circuit}}(t) \tag{10}$$

The signal $m_{\text{Jerk\_circuit}}(t)$ is one of the parameters of equation (10). After finishing the encryption process the original signal can be recovered with the following procedure.

$$m_{\text{New\_Signal}}(t) = m_{\text{encryption}}(t) - m_{\text{Jerk\_circuit}}(t) \tag{11}$$

So, $m_{\text{New\_Signal}}(t)$ is the original signal and must be the same with $m_s(t)$. Due to the fact that the input signal can be recovered from the output signal, it turns out that it is possible to implement a secure communication system using the proposed chaotic system.

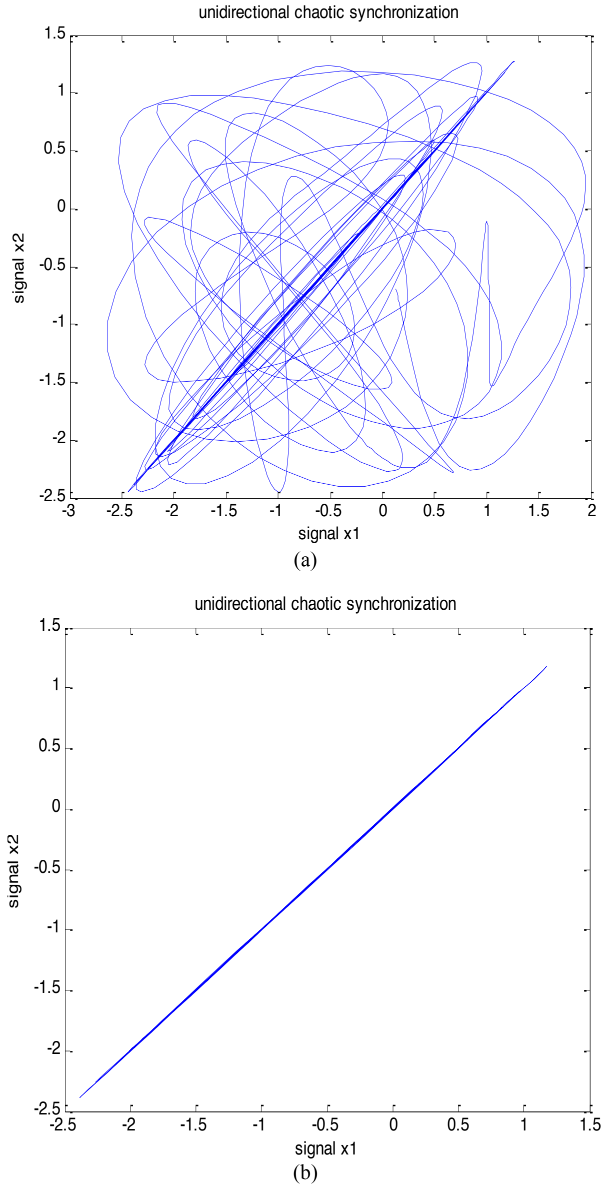


Fig. 8. Simulation phase portrait in x_2 versus x_1 plane, in the case of unidirectional coupling, for (a) ξ = 0.184 (full desynchronization) and (b) ξ = 0.25 (full synchronization), with MATLAB 2010.

4.1. Numerical Simulations Using MATLAB

Figs.9(a)-(c) show the numerical simulation results with MATLAB 2010 for the proposed chaotic masking communication scheme, for A = 1V and f = 4KHz. The chosen value of the coupling factor is ξ = 0.250 in order that the system be in a chaotic synchronization regime.

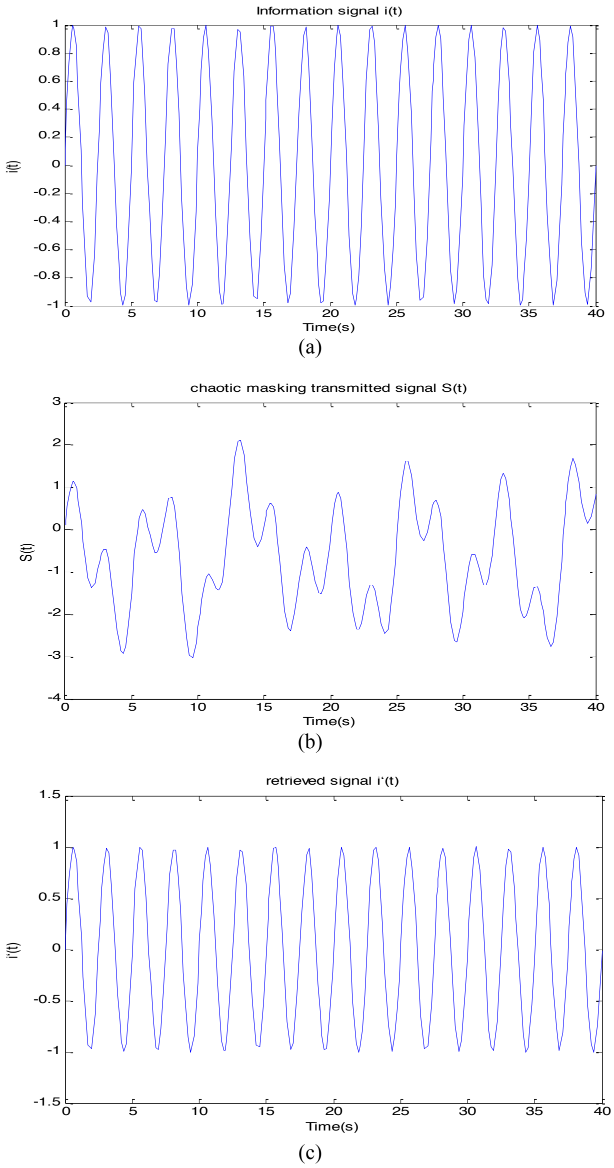
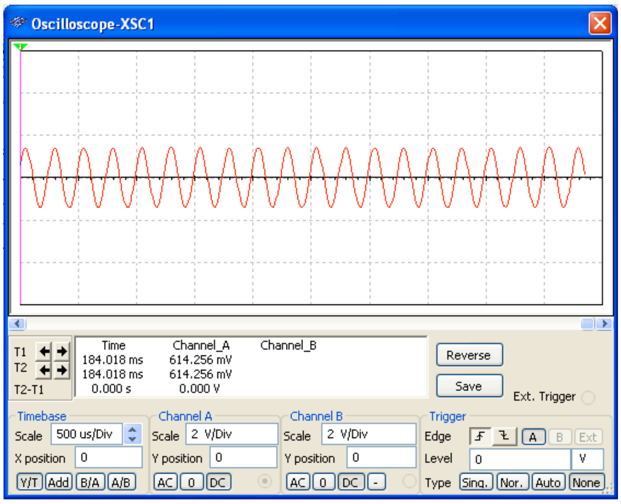


Fig. 9. MATLAB 2010 simulation of Jerk circuit masking communication system, for A = 1V and f = 4KHz: (a) Information signal, (b) Chaotic masking transmitted signal, (c) Retrieved signal.

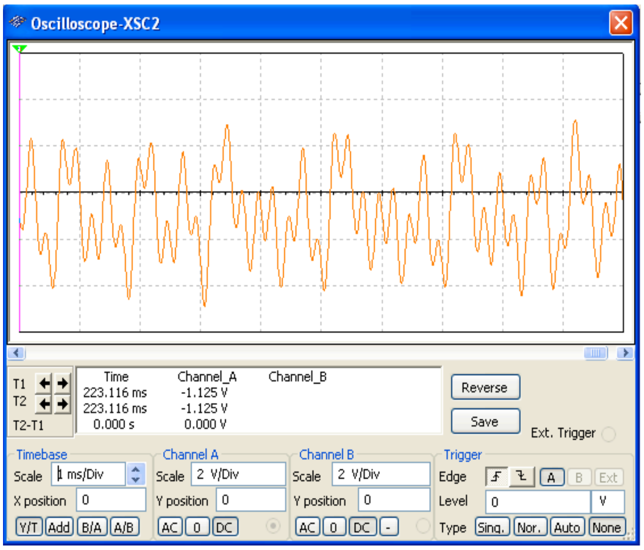
4.2 Analog Circuit Simulation Using MutiSIM 10.0

In chaos-based secure communication scheme, chaos synchronization is the critical issue, because the two identical chaos generators in the transmitter and the receiver end need to be synchronized. Information signal is added to the chaotic signal at transmitter and at receiver the masking signal is regenerated and subtracted from the receiver signal. For synchronization of transmitter and receiver, unidirectional coupling method of full synchronization technique is used. Fig.10 shows the MultiSIM 10.0 simulation results for the masking signal communication system, for A = 1V, f = 4KHz and ξ = 0.250.

Also, in the proposed masking scheme, the sinusoidal wave signal of amplitude 1V and frequency 4KHz is added to the synchronizing driving chaotic signal in order to regenerate the original driving signal at the receiver. Thus, as it can be shown from Fig.10(c), the message signal has been perfectly recovered by using the signal masking approach through the synchronization of chaotic Jerk circuits. Furthermore, simulation results with Multisim 10.0 have shown that the performance of chaotic Jerk circuits in chaotic masking and message recovery is very satisfactory. Finally, Figure 11 shows the circuit schematic of implementing the Jerk circuit chaotic masking communication scheme.



(a)



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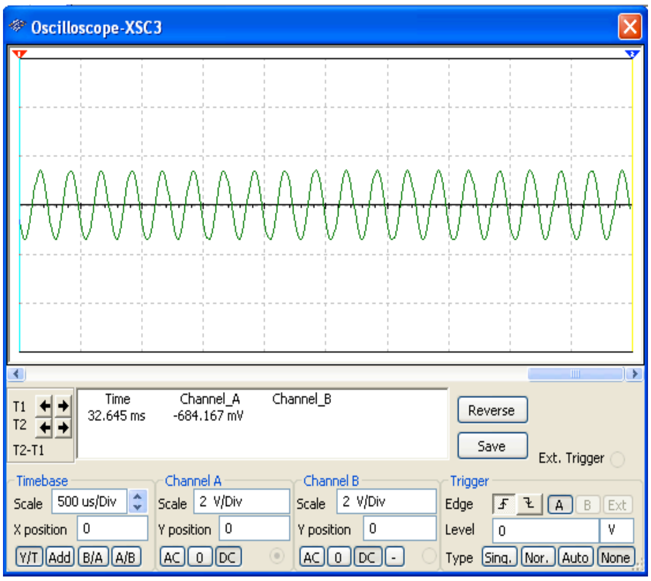

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Fig.10. MultiSIM 10.0 outputs of Jerk circuit masking communication systems, for A = 1V and f = 4KHz: (a) Information signal, (b) Chaotic masking transmitted signal, (c) Retrieved signal.

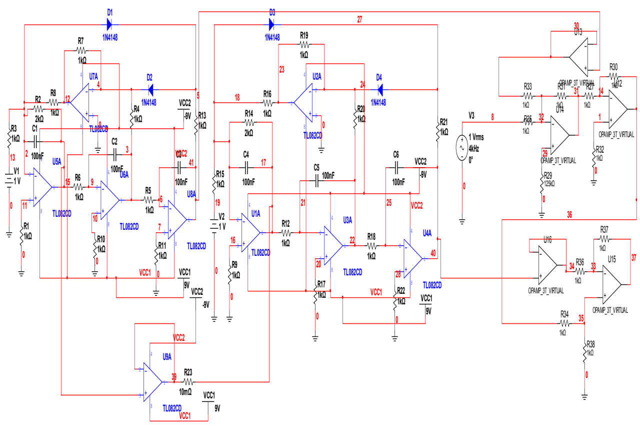

Fig.11. Jerk's circuit masking communication system.

5. Conclusion

In this paper, the full synchronization of unidirectionally coupled Jerk circuits has been investigated. We have demonstrated, by simulations, that chaotic circuits can be synchronized and used in a secure communication scheme. Chaos synchronization and chaos masking were realized by using MATLAB 2010 and MultiSIM 10.0 programs. Furthermore, comparisons are made with existing results. Finally, the simulation results demonstrate the effectiveness of the proposed scheme.

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