

## Modeling Surface Characteristics of Finished Parts

R. G. Kudoyarov, S. I. Fetsak and Yu. V. Idrisova\*

Ufa State Aviation Technical University, Ufa, Russian Federation

Received 25 September 2014; Accepted 30 September 2014

### Abstract

This article describes a method to estimate the effect of dynamic processes that occurring in an elastic system of a machine on waviness and roughness of the surface of a product.

*Keywords:* spectrum of external perturbation, elastic system of equipment, cutting action, surface waviness, surface roughness

### 1. Introduction

The quality of a mechanical engineering product depends on the condition of equipment, the dynamic processes in the elastic system of the equipment and cutting parameters. Surface waviness and roughness of a mechanical engineering product are results of defects of equipment and wear of them.

High-speed machining has certain limitations such as a maximum precision and a throughput of equipment. This characteristics can be achieved with certain operations of a manufacturing method in combination with certain stock, tool, machining rate and computer numerically controlled machine. A high throughput and good surface quality of a product are possible due to a wear control system and a quality forecasting system of a workpiece in relation to the dynamic processes of a machine.

The efficiency of this equipment can be increased with a model which estimate vibrations of operating parts of a CNC machine, dynamic features of equipment, possible manufacturing errors, errors of mounting, an impact of defects on a surface a product. This can be necessary in multi-axes machines[1, 2].

The basis of the method developed is a hypothesis that disturbances occurring in the course of the operation of a machine, result in vibration of a tool and the work affecting the surface quality of manufactured parts. The ratio between surface errors and vibrations of a tool and a workpiece can be found by analyzing the shaping process.

### 2. The model structure to determine the waviness and roughness of a product surface

The algorithm for creating a model of waviness and roughness of the product surface has a modular structure in which each module is designed for specific tasks, which include:

- forming of spectrum of external perturbation Eqs. (1);
- modeling of dynamic characteristics of elastic system of equipment Eqs. (3)
- determining dynamic characteristics of cutting Eqs. (2)
- forming of forced vibrations of operating parts of a machine (free movement with recording dynamic characteristics of cutting) Eqs (4)

The block diagram of the method is shown in Figure 1.

#### 2.1. Forming of the spectrum of external perturbation

The main reasons for external disturbance in a metal cutting machine are:

- imbalance of the spindle and drive parts;
- manufacturing and assembly errors of bearings, antifriction slideways, rolling screw motion drive;
- manufacturing and assembly errors of motor parts

These defects of equipment are sources of external disturbances. The main parameters of external vibration are amplitude and frequency. Mathematical models of these (1)

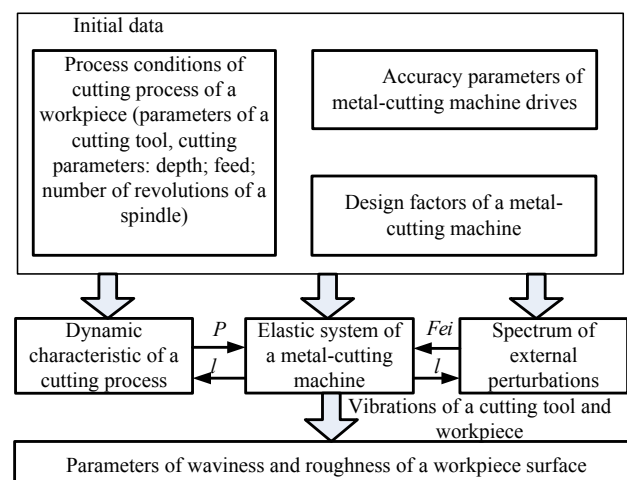


Fig. 1. Block diagram of the method to determine the waviness and roughness of a surface of a product.

\* E-mail address: Yu.V.Idrisova@mail.ru

sources are presented in the form of oscillation amplitude-frequency spectra of the Fourier series expressions of disturbing forces.

$$D_r^s = \sum_{i=1}^n \Delta p_r^s \cos \omega_i \quad (1)$$

where  $p_r$  represents the amplitude of external disturbances, N;  $\omega$  is the frequency of external disturbances, Hz;  $r$  is a harmonic number;  $s$  is the error type.

### 2.2. Dynamic characteristics of cutting

The cutting process is a part of dynamical system of a metal cutting machine, the input parameter of cutting is relative vibration of a machine, an output parameter is the cutting force acting on the elastic system of a metal cutting machine. Relative oscillation of a working machine causes a change in the cutting force and leads to a reduction in the processing precision.

Physical phenomena that characterize the process of chip forming lead to a shift in the time (lag) in changes of cutting force  $P$  with respect to cause this change in the relative location of the tool and workpiece  $h$ , thus cutting process is of the inertia characteristic of the first order.

Metal processing involves a multiple bit cutter, with each subsequent blade located behind the previous one. In this case, the transfer function of cutting will be determined by this expression (2) [3].

$$W_p = \frac{P}{t} = \frac{K_p(1 + \exp(\tau \cdot p))}{T_p p + 1} \quad (2)$$

where  $T_p$  – the time constant of chip forming, sec;  $K_p$  – a cutting value, which depends on the cutting force, cutting parameters and type of treatment;  $\tau$  – lag factor, which depends on number of tool blades,  $\tau = t/Z$ , where  $t$  – depth of cut,  $Z$  – number of blades.

### 2.3. Modeling the dynamic characteristics of an elastic system of a metal cutting machine

To create a dynamic system, a real system is replaced by a computational scheme, i.e. a system with a finite number of degrees of freedom. A computational scheme has to be equivalent to the real system of a metal cutting machine with a certain accuracy, as well as has to be simple with a minimum number of lumped masses.

A computational scheme is a scheme which has lumped masses which are connected by weightless elastic and dissipative elements with linear characteristics. This simplification is used as most parts of a metal cutting machine have big mass, are solid bodies and their deformations appear mostly in junctions.

To form a mathematical model of an elastic system it is necessary to consider the forces which describe the working processes in the elastic system of a metal cutting machine.

### 2.4. Forming a model of forced oscillations metal cutting machine's elements

The set of second order differential equations that describes relative oscillations of a tool and a workpiece, has  $n$  equations,  $n$  is a number of arrays, for array with number  $i$  balance equation has following form:

$$m_i \ddot{l}_i + h_i \dot{l}_i + c_i l_i = \sum_{i=1}^n \bar{F}_{Bi} \quad (3)$$

where  $l_i$  – movement of a node, in the X, Y or Z axis.

The left part of each equation (3) represents an action of internal forces and inertia in the  $i$ -th element of a metal cutting machine, the right part of each equation represents the action of external forces  $F_e$  in  $i$ -th element.

Depending on the type of processing the working movement of a metal-cutting machine will vary. This applies that the normal to the surface of a workpiece changes its direction in the coordinate system of a metal cutting machine.

It should also be considered that the appropriate type of interpolation in the processing of complex surfaces should be selected. In this case, the amplitude of the resultant vibration vector of a tool and the vector vibrations of a workpiece should be obtained for the displacement actuator according to the block diagram of a metal cutting machine.

To determine the absolute amplitudes of the oscillations of  $n$ -th node design scheme of a metal cutting machine it is necessary to divide the entire range of investigated frequencies into sections  $\Delta f$ . The RMS amplitude of vibration displacement on the  $j$ -th coordinate of the displacement vector in  $\bar{H}$  band width  $\Delta f$  equals:

$$A_j(\Delta f) = \sqrt{\sum_s \sum_{\Delta f} H_{kj}^2} \quad (4)$$

where  $s$  is the number of vibration sources.

The amplitudes of the  $k$ -th harmonic of the  $j$ -th position in the frequency band  $\Delta f$  vibration vector of a metal cutting machine can be found in their relative amplitude of oscillations in the same frequency range  $\Delta f$ . Relative displacements of parts of a metal cutting machine are considered only in a plane normal to the surface of a workpiece. Oscillation amplitude can be represented as the sum of the mean square amplitudes of the  $k$ -th harmonic of the  $j$ -th coordinates of the vectors of parts of a metal cutting machine vibration

$$A_{rel,kj}(\Delta f) = \sqrt{\sum_s \sum_{\Delta f} (H_{tool,kj}^2 + H_{workpiece,kj}^2)} \quad (5)$$

where  $H_{tool,kj}^2$  and  $H_{workpiece,kj}^2$  are values characterizing the amplitude of the  $k$ -th harmonic  $j$ -th coordinate vectors of vibration tool and a workpiece, respectively.

### 2.5. Determination of the waviness and roughness parameters of parts

The relationship between waviness and roughness of a product and vibrations of a tool and a workpiece can only be found by analyzing the forming process of a product. The parameters of waviness and roughness of a product surface are the result of combination of the following copying the tool edges; relative vibrations of the a tool and a workpiece; the previous waveform of distortion on a workpiece surface during forming of the subsequent one (treatment on the trail).

The decisive factor in waviness and roughness of a surface, is the ratio of the oscillation frequency of a tool and a workpiece  $f_i$  to the number of workpiece revolutions  $n$  [4]. If the above ratio is an integer, the vibrations affect the

forming errors on a cross-sectional surface. The vibrations of a tool and a workpiece directly transfer to the working surface. If the ratio is not an integer, it can be represented as the sum of a fractional part and integer part. In this case, there is a phase shift that happens in the cross section of a workpiece every revolution. In the absence of vibrations of a tool and a workpiece the waviness and roughness of a workpiece surface are the result of the geometrical shape of the cutting tool.

The deviation of the product profile (the result of workpiece machining) can be characterized by a set of variations with different frequencies, to determine the spectrum of phase angles, amplitudes and frequencies. Therefore the spectrum of deviation of a product profile can be considered as a harmonic series with a period of expansion equal to twice the length of the individual parts of a product surface area [5].

The total error of a product surface profile is:

$$\delta_{\Sigma}(\tau) = \frac{A}{2} + \sum_{i=1}^m \frac{A_i \sin i\pi z}{2l} \quad (6)$$

where  $A$  and  $A_i$  are deviations of the size and amplitude of  $i$ -th harmonic shape;  $l$  is the length of an elementary surface areas of a product;  $z$  is the current coordinate;  $i$  is the serial number of the harmonic.

Vibrations of a tool and a workpiece can also be represented as a harmonic series with the same period of expansion.

$$A_{\Sigma}(\tau) = \frac{A'}{2} + \sum_{i=1}^m \frac{A'_i \sin i\pi z}{2l} \quad (7)$$

Waviness and roughness are determined by using linear models of machine accuracy and a variation function of formation [7]. The shaping function is an analytic dependence that link movements of shaping parts of a metal-cutting machine with the trajectory of points of a tool relative to a workpiece in the workpiece coordinate system. The composition of the machine system includes the support system and driving elements.

To obtain estimates of waviness and roughness of a machined surface there is need to build a nominal reference surface [7]. The base surface has the same shape as the nominal surface, and is determined by the points of the real surface in such a way that the mean square distance between the base points of the real surface is minimum [7]. The sequence of steps to calculate waviness and roughness of the machined surface is shown in Fig. 2.

To determine waviness and roughness it is necessary to determine the height of the peak of a wave and the depth of the trough of a wave in the  $j$ -th section of a product using expressions

$$\Delta \tilde{y}_{ij}^{tr} = \frac{1}{4} \sum_{i=1}^n \frac{4A_{y_i} R}{S_z^2} \sin(k_j^{p_{mj}} \varphi_i + \alpha_i), \quad (8)$$

$$\Delta \tilde{y}_{ij}^{cr} = \Delta \tilde{y}_{ij}^{tr} + \frac{1}{2} (\tilde{Q}_{1,2}^j)^2,$$

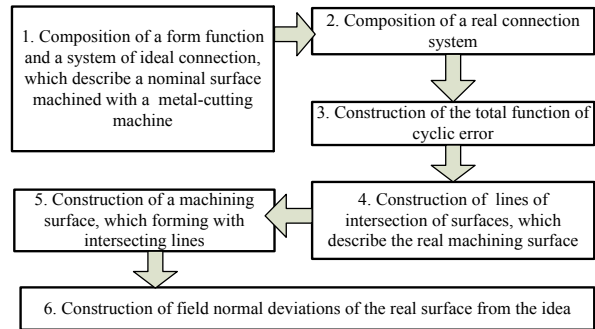


Fig. 2. Block diagram of calculating waviness and roughness of a machined surface

where  $k_j^{p_{mj}} = (P_{mj} - 1) + (j - 1) / N$ ,  $n$  is a number of sections of the surface;  $A_y$  is the amplitude of the relative vibration of working parts of a metal-cutting machine;  $\tilde{Q}_{1,2}^j$  is an angle between two neighboring tracks at the desired point of intersection;  $\varphi_i$  is an angle defining the position of the blade on the surface and the polar angle is equal to the  $i$ -th point of the blade.

### 3. Calculating the profile surface of a workpiece

The calculation of these quality parameters of a product (waviness and roughness) was made with the proposed method Eqs (1)-(7) for a product surface milled with a metal cutting machine model 500VS (Fig. 3).

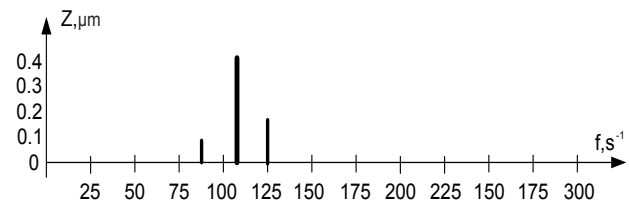


Fig. 3. Calculated amplitude-frequency spectrum of irregularity of the product surface

The source data used: material of the workpiece GG20, tool: end mill diameter  $D = 38$  mm, number of teeth  $z = 4$ , the cutting parameters: cutting depth  $t = 0,1$  mm, feed  $S = 200$  mm / min, spindle speed  $600$  rpm<sup>-1</sup>.

The relation between the frequency of occurrence of waviness and surface roughness of a machined product with the distance between two identical waves on the surface of a product is defined by  $f_H = 60\Delta L / S$ , where  $S$  is the feed, mm / min;  $\Delta L$  is the length of a treated surface.

Comparison between the calculated profile of a workpiece surface with the measured one of a product (Fig. 4) is characterized by a relative deviation of 5%, that proves the correctness of the proposed mathematical equations to determine the quality of machined products.

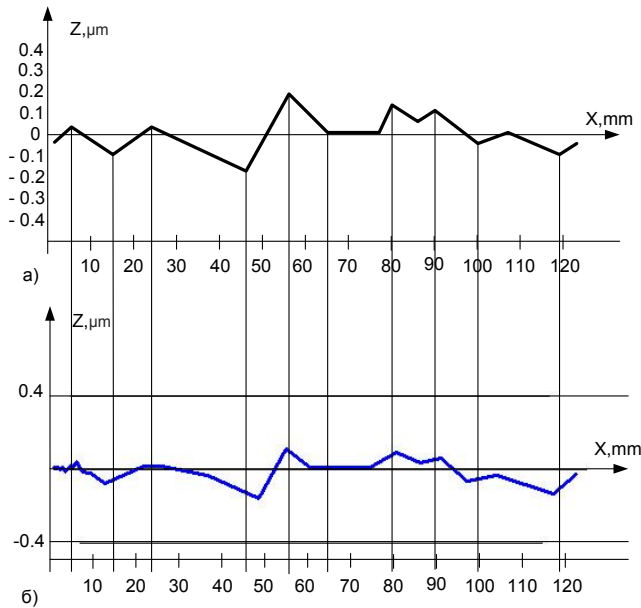


Fig. 4. Profile of the product surface : a – calculated (1)-(8), b – measured.

#### 4. Conclusions

1. A method was developed to assess the impact of the dynamic processes occurring in the elastic system of a metal cutting machine on the waviness and roughness of the surface of a product, which takes into consideration the principles of shaping its surface at the current condition of equipment and processing characteristics at high frequencies (speeds) of spindle rotation.

2. The method involves modeling the relative vibration of a tool and a workpiece and the calculation of the theoretical values of surface roughness of a product. Waviness and roughness parameters are determined using linear models of a metal cutting machine accuracy and variation of function of formation.

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