

## Adaptive Synchronization of Memristor-based Chaotic Neural Systems

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### Abstract

Chaotic neural networks consisting of a great number of chaotic neurons are able to reproduce the rich dynamics observed in biological nervous systems. In recent years, the memristor has attracted much interest in the efficient implementation of artificial synapses and neurons. This work addresses adaptive synchronization of a class of memristor-based neural chaotic systems using a novel adaptive backstepping approach. A systematic design procedure is presented. Simulation results have demonstrated the effectiveness of the proposed adaptive synchronization method and its potential in practical application of memristive chaotic oscillators in secure communication.

**Keywords:** Adaptive synchronization, memristors, chaos, chaotic systems, backstepping.

### 1. Introduction

The memristor, defined by the relationship between the flux and the charge of a device, was theoretically predicted by Leon Chua in 1971 and called the fourth fundamental circuit element after the resistor, the capacitor and the inductor [1]. In 2008, Williams and his team from HP Lab proved the existence of the memristor in nanoscale electronics while developing ultra-high density nonvolatile memory [2]. Afterwards, the research on memristors or memristive systems has gained ever-increasing attention from both academia and industry [3-12]. In particular, many efforts have been devoted into discovery of some important properties of typical memristors [3,4], various memristive devices and materials [5-7], as well as promising application potentials [8-14].

Due to those pioneers' valuable work, the key features of the memristor can be summarized as follows.

- (i) The memristor is a kind of nonlinear devices in simple sandwiching structure, featuring hysteretic current-voltage characteristic under periodic external excitation conditions.
- (ii) The memristor's capabilities of nanoscale size, variable resistance and power-off mode storage make it a competitive candidate of the next-generation nonvolatile memory [8,9].
- (iii) The conductivity of a memristor depends on the total flux/charge ever passing through it. This property is very similar to the biological synaptic plasticity, that is,

the strength of a synaptic weight is in the control of the ionic flowing through the synapse between two adjacent neurons. Thereby, by combining the advantages of tiny scale and simple structure, the memristor naturally becomes the preferred artificial synapses in large-scale and massively-parallel neuromorphic architectures that merge computation and memory [10-11].

- (iv) The memristor also has potential in nonlinear circuit design and realization such as chaotic oscillators.

A novel implementation scheme for chaotic oscillators using nanoscale memristors might achieve richer dynamic behaviors with much smaller and simpler circuits, compared with the traditional operational-amplifier-based method. In fact, many memristive chaotic systems have been designed and investigated [12-14]. The memristive element used in most of these systems is the generalized memristor or memristive system with an odd-symmetric flux-charge characteristic similar to the current-voltage curve of Chua's diode [12]. Recently, more attention has been paid on chaotic systems consisting of HP memristors [13,14]. In this paper, the latter will be focused on.

Since the chaos synchronization was shown to be possible by Pecora and Carroll [15], synchronization between coupled chaotic systems has been extensively investigated [16-20]. The concept of chaos synchronization refers to making two identical chaotic dynamical systems with different initial conditions oscillate in a synchronized manner [20]. Up to now, various synchronization phenomena have been observed in different chaotic systems, including complete synchronization, generalized synchronization, phase synchronization, lag synchronization and so on [16]. In practical applications, it is well known that synchronization plays an essential role in

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chaos secure communication systems. So far, for the canonical chaotic systems such as Lorenz's system, Chua's circuits, and Chen's system, many different synchronizing approaches have been proposed and intensively studied, including conventional linear control schemes and advanced nonlinear control techniques [18]. Recently, Fernando Corinto has also demonstrated the influence of the memristor synapse on the synchronous behaviors of two Hindmarsh-Rose chaotic neurons [21] and two FitzHugh-Nagumo chaotic neurons [22].

In many real applications, the parameters of chaotic systems under study might not be exactly known. In such cases, the so-called adaptive controllers are suitable. As a powerful adaptive control scheme, backstepping has been widely used in applications since it can guarantee tracking, global stability, and transient performance of a broad class of strict-feedback systems [20]. Unfortunately, very few discussions on synchronization of the memristor-based chaotic systems with parametric uncertainties have been reported, which motivates this study.

This paper aims at studying adaptive synchronization of two coupled memristor-based chaotic neural systems using a backstepping approach and presenting a systematic design procedure. The rest of the paper is organized as follows. In Section 2, a third-order memristor-based chaotic neural system is firstly introduced. Then, a controller for synchronization of the two coupled memristor-based chaotic neural systems is designed. Adaptive synchronization of the chaotic neural systems with uncertain parameters is studied in Section 3. Simulation results are presented in Section 4 and Section 5 outlines the conclusions.

## 2. Synchronizing Two Coupled Memristor-Based Chaotic Neural Systems via a Backstepping Design

### 2.1. The Memristor-based Chaotic System

The memristor-based chaotic neural system is described as follows [13],

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = -ax_1 + x_3 \\ \dot{x}_3 = 1 + x_2 + bg(-|x_1|) \end{cases} \quad (1)$$

where  $x_1$ ,  $x_2$  and  $x_3$  are the state variables;  $a$  and  $b$  are positive constant parameters;  $g(\cdot) = 1000f(\cdot)$  and  $f(\cdot)$  denotes the charge flowing through the memristor, precisely expressed by

$$f(x) = \begin{cases} \frac{x - c_1}{R_{OFF}}, & x < c_3 \\ \frac{\sqrt{2kx + R_0^2} - R_0}{k}, & c_3 \leq x < c_4 \\ \frac{x - c_2}{R_{ON}}, & x \geq c_4 \end{cases} \quad (2)$$

In the above equation,  $x$  indicates the flux across the memristor;  $R_{ON}$  and  $R_{OFF}$  are the lower and upper limit value of the memristance (memristor resistance),

respectively;  $k = \frac{(R_{ON} - R_{OFF})\mu_V R_{ON}}{D^2}$  is a predefined

constant;  $\mu_V = 10^{-14} m^2 s^{-1} V^{-1}$  refers to the average mobility of the dopants;  $D$  is the size of the  $TiO_2/TiO_{2-x}$  layers;  $R_0$  represents the initial value of the memristance; and  $c_1$ ,  $c_2$ ,  $c_3$  and  $c_4$  are constants for the sake of simplification of the formula, as below:

$$\begin{aligned} c_1 &= -\frac{(R_{OFF} - R_0)^2}{2k}; \quad c_2 = -\frac{(R_{ON} - R_0)^2}{2k}; \\ c_3 &= \frac{R_{OFF}^2 - R_0^2}{2k}; \quad c_4 = \frac{R_{ON}^2 - R_0^2}{2k} \end{aligned}$$

A set of typical parameters that can generate chaotic dynamics is:  $a = 10$  and  $b = 10$ . With the initial condition set to be  $[0, 1, 0]$ , this system yields a chaotic attractor, as shown in Fig. 1. The corresponding time evolutions of the states are illustrated in Fig. 2.

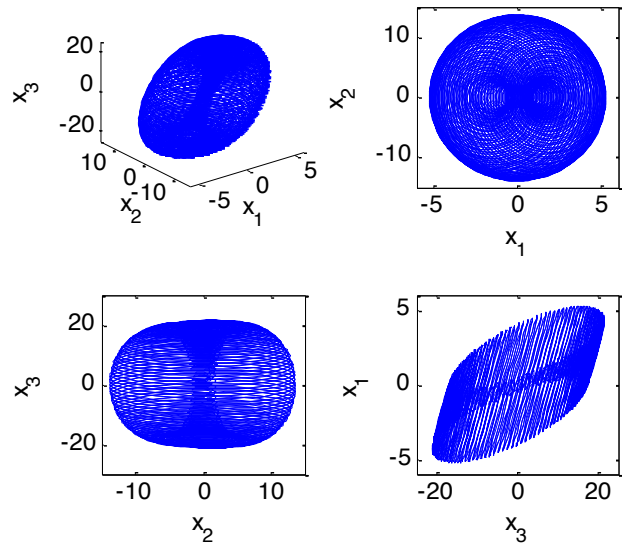


Fig. 1. Chaotic attractor generated in system (1).

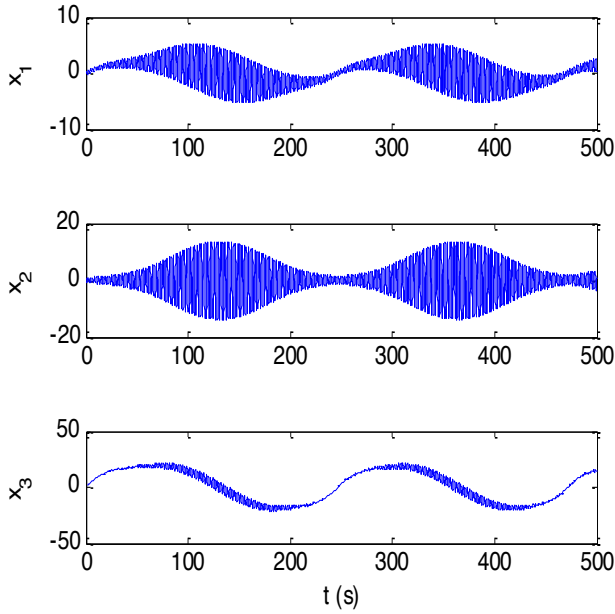
### 2.2. Synchronization of the Memristor-based Chaotic Neural Systems via a Backstepping Design

Recall the definition of chaos synchronization [20]. For two chaotic neural systems,

$$\dot{x} = \hat{f}(t, x) \quad (3)$$

$$\dot{y} = \hat{g}(t, y) + u(t, x, y) \quad (4)$$

where  $x, y \in \mathbb{R}^n$ ,  $\hat{f}, \hat{g} \in C^r[\mathbb{R}_+ \times \mathbb{R}^n, \mathbb{R}^n]$ ,  $u \in C^r[\mathbb{R}_+ \times \mathbb{R}^n \times \mathbb{R}^n, \mathbb{R}^n]$ ,  $r \geq 1$ , and  $\mathbb{R}_+$  is the set of non-negative real numbers, assume that (3) is the drive system and (4) is the response system,  $u(t, x, y)$  is a controller, then these two systems are said to be synchronized if  $\forall x(t_0), y(t_0) \in \mathbb{R}^n$ ,  $\lim_{t \rightarrow \infty} \|x(t) - y(t)\| = 0$ .



**Fig. 2.** Time evolutions of states  $x_1$ ,  $x_2$ ,  $x_3$  of chaotic system (1).

In this section, the objective is to design a controller  $u_1$  such that the controlled memristor-based chaotic neural system, that is, the response system,

$$\begin{cases} \dot{y}_1 = y_2 \\ \dot{y}_2 = -ay_1 + y_3 \\ \dot{y}_3 = 1 + y_2 + bg(-|y_1|) + u_1 \end{cases} \quad (5)$$

is synchronous with the drive system codified by (1). In other words, we aim at making the dynamical error system between the drive system (1) and the response system (5),

$$\begin{cases} \dot{e}_1 = e_2 \\ \dot{e}_2 = -ae_1 + e_3 \\ \dot{e}_3 = e_2 + b(g(-|y_1|) - g(-|x_1|)) + u_1 \end{cases} \quad (6)$$

asymptotically stable, where  $e_1 = y_1 - x_1$ ,  $e_2 = y_2 - x_2$ ,  $e_3 = y_3 - x_3$ . Next, the backstepping procedure [17] is used to design the controller  $u_1$ .

The first partial Lyapunov function is chosen as,

$$V_1 = \frac{1}{2} z_1^2 \quad (7)$$

where  $z_1 = e_1$ . Its derivative along the solutions of systems (1) and (5) is

$$\dot{V}_1 = z_1 \dot{z}_1 = -z_1^2 + z_1(e_1 + e_2) \quad (8)$$

Then, the second partial Lyapunov function is given by,

$$V_2 = V_1 + \frac{1}{2} z_2^2 = \frac{1}{2} (z_1^2 + z_2^2) \quad (9)$$

where  $z_2 = e_1 + e_2$ . Similarly, its derivative along the solutions of systems (1) and (5) can be obtained as,

$$\dot{V}_2 = -z_1^2 - z_2^2 + z_2((2-a)e_1 + 2e_2 + e_3) \quad (10)$$

Thus, we can form the Lyapunov function,

$$V = V_2 + \frac{1}{2} z_3^2 = \frac{1}{2} (z_1^2 + z_2^2 + z_3^2) \quad (11)$$

where  $z_3 = (2-a)e_1 + 2e_2 + e_3$ . Its time derivative along the solutions of systems (1) and (5) is calculated as,

$$\begin{aligned} \dot{V} = & -z_1^2 - z_2^2 - z_3^2 + z_3((3-3a)e_1 + (6-a)e_2 \\ & + 3e_3 + b(g(-|y_1|) - g(-|x_1|)) - u_1) \end{aligned} \quad (12)$$

Now, we have the following theorem.

**Theorem 1.** If the controller  $u_1$  is designed as:

$$\begin{aligned} u_1 = & (3a-3)e_1 + (a-6)e_2 - 3e_3 - \\ & b(g(-|y_1|) - g(-|x_1|)) \end{aligned} \quad (13)$$

then the controlled memristor-based system (5) is globally synchronous with the drive system (1).

*Proof.* Substituting (13) into (12), we have

$$\dot{V} = -z_1^2 - z_2^2 - z_3^2 \leq 0 \quad (14)$$

Thus  $V$  is positive and  $\dot{V}$  is semi-negative. But, we cannot immediately conclude that the origin of error system (6) is asymptotically stable. In fact, as  $V$  is a positive and decrescent function with  $\dot{V} \leq 0$ , then  $e_1, e_2, e_3 \in \zeta_\infty$ , that is, they are bounded. In addition, from (14) we can show that the square of the error signals  $e_i$  ( $i=1,2,3$ ) is integrable with respect to time, i.e.,  $e_1, e_2, e_3 \in \zeta_2$ . Since all of the components on the right-hand side of the error system (6) are bounded, we can easily see that  $\dot{e}_1, \dot{e}_2, \dot{e}_3 \in \zeta_\infty$  under any initial conditions. Finally, based on Barbalat's lemma, we have  $\lim_{t \rightarrow \infty} \|e(t)\| = 0$ . Therefore, the controlled memristor-based chaotic system (5) is globally synchronous with the system (1). The proof is thus completed.

### 3. Adaptive Synchronization of Memristor-based Chaotic Neural Systems with Uncertain Parameters

In the synchronization method presented in Section 2, it is assumed that the system parameters are known a priori. However, as mentioned before, in many practical cases, it is not easy to get the exact value of the system parameters. In such cases an adaptive controller is needed to synchronize these chaotic neural systems. In this section, we present an adaptive approach to synchronizing the memristor-based chaotic neural systems with one or two unknown parameters using the backstepping procedure.

#### 3.1. With Unknown Parameter $b$

Assuming parameter  $b$  is unknown, the controlled memristor-based chaotic neural system is represented by

$$\begin{cases} \dot{y}_1 = y_2 \\ \dot{y}_2 = -ay_1 + y_3 \\ \dot{y}_3 = 1 + y_2 + bg(-|y_1|) + u \end{cases} \quad (15)$$

Then the corresponding dynamical error system is described as follows,

$$\begin{cases} \dot{e}_1 = e_2 \\ \dot{e}_2 = -ae_1 + e_3 \\ \dot{e}_3 = e_2 + b(g(-|y_1|) - g(-|x_1|)) + u \end{cases} \quad (16)$$

To design the controller  $u$ , we firstly choose a partial Lyapunov function,

$$V_1 = \frac{1}{2}z_1^2 \quad (17)$$

where  $z_1 = e_1$ . Its derivative along the solutions of systems (1) and (15) is

$$\dot{V}_1 = z_1\dot{z}_1 = -z_1^2 + z_1(e_1 + e_2) \quad (18)$$

Next, we select the second partial Lyapunov function as follows,

$$V_2 = V_1 + \frac{1}{2}z_2^2 = \frac{1}{2}(z_1^2 + z_2^2) \quad (19)$$

where  $z_2 = e_1 + e_2$ . Though some calculations, we can get its time derivative,

$$\dot{V}_2 = -z_1^2 - z_2^2 + z_2((2-a)e_1 + 2e_2 + e_3) \quad (20)$$

Now, we form the Lyapunov function,

$$\begin{aligned} V &= V_2 + \frac{1}{2}z_3^2 + \frac{1}{2}(\hat{b} - b)^2 \\ &= \frac{1}{2}(z_1^2 + z_2^2 + z_3^2) + \frac{1}{2}(\hat{b} - b)^2 \end{aligned} \quad (21)$$

where  $z_3 = (2-a)e_1 + 2e_2 + e_3$ , and  $\hat{b}$  is the estimate of the unknown parameter  $b$ . The time derivative of the Lyapunov function can be expressed as follow,

$$\begin{aligned} \dot{V} &= -z_1^2 - z_2^2 \\ &\quad + z_3[(1-2a)e_1 + (4-a)e_2 + u + \\ &\quad b(g(-|y_1|) - g(-|x_1|))] + (\hat{b} - b)\dot{\hat{b}} \end{aligned} \quad (22)$$

We then get the following theorem.

**Theorem 2.** If the controller  $u$  is designed as:

$$u = (3a-3)e_1 + (a-6)e_2 - 3e_3 - \hat{b}(g(-|y_1|) - g(-|x_1|)) \quad (23)$$

and the parameter adaptive law of  $\hat{b}$  is designed as:

$$\dot{\hat{b}} = ((2-a)e_1 + 2e_2 + e_3)(g(-|y_1|) - g(-|x_1|)) \quad (24)$$

Then the controlled response system (15) is globally synchronous with the drive system (1).

*Proof.* Substituting (23) and (24) into (22), we have

$$\dot{V} = -z_1^2 - z_2^2 - z_3^2 \leq 0 \quad (25)$$

So  $V$  is positive define and  $\dot{V}$  is negative semi-definite. Recall the similar arguments presented in the proof of Theorem 1 in Section 2, we have that all the signals in both systems are bounded,  $\lim_{t \rightarrow \infty} z_i = 0$  ( $i=1, 2, 3$ ), and

$\lim_{t \rightarrow \infty} e_i = 0$  ( $i=1, 2, 3$ ). Therefore, the adaptive

synchronization of two coupled memristor-based chaotic neural systems (1) and (15) with an unknown parameter is achieved. The proof is thus completed.

**Remark 1:** It should be noted that that the estimated parameter  $\hat{b}$  will approach a constant and is bounded. However, there is no guarantee that it will approach its true value  $b$ .

### 3.2 With Unknown Parameters $a$ and $b$

Similarly, if parameters  $a$  and  $b$  are both unknown, we consider the following controlled response chaotic neural system

$$\begin{cases} \dot{y}_1 = y_2 \\ \dot{y}_2 = -ay_1 + y_3 + u_1 \\ \dot{y}_3 = 1 + y_2 + bg(-|y_1|) + u_2 \end{cases} \quad (26)$$

By choosing the following Lyapunov function,

$$V = \frac{1}{2}(z_1^2 + z_2^2 + z_3^2) + \frac{1}{2}(\hat{a} - a)^2 + \frac{1}{2}(\hat{b} - b)^2 \quad (27)$$

where  $z_1 = e_1$ ,  $z_2 = e_1 + e_2$ , and  $z_3 = e_3$ ,  $\hat{a}$  and  $\hat{b}$  are the estimates of unknown parameters  $a$  and  $b$  respectively, we have the following theorem.

**Theorem 3.** If  $u_1$  and  $u_2$  are designed as,

$$\begin{aligned} u_1 &= (\hat{a} - 2)e_1 - 2e_2 - e_3 \\ u_2 &= -e_2 - e_3 - \hat{b}(g(-|y_1|) - g(-|x_1|)) \end{aligned} \quad (28)$$

and the parameter adaptive laws of  $\hat{a}$  and  $\hat{b}$  are designed as follows,

$$\begin{aligned} \dot{\hat{a}} &= -(e_1 + e_2)e_1 \\ \dot{\hat{b}} &= e_3(g(-|y_1|) - g(-|x_1|)) \end{aligned} \quad (29)$$

then the controlled uncertain memristive chaotic neural system (26) is globally synchronous with system (1). The proof is similar to that of the Theorem 2, and thus omitted.

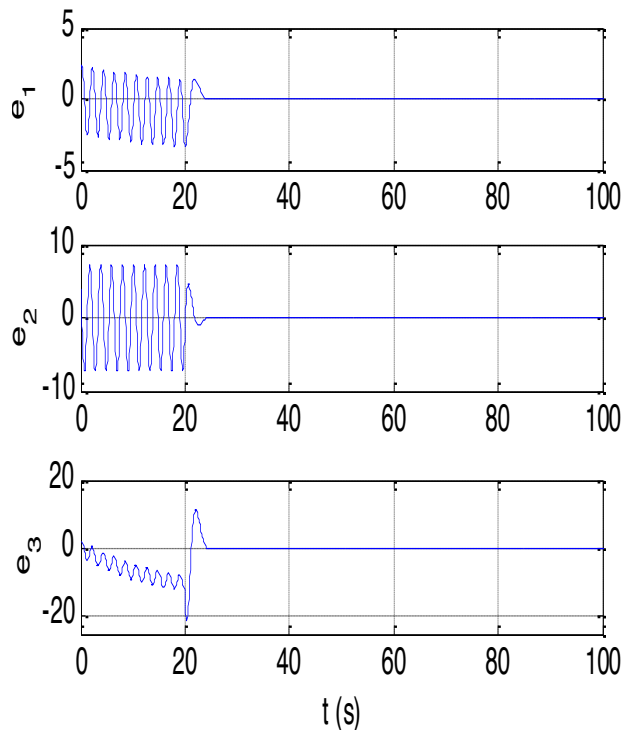
#### 4. Simulation Results

The numerical simulations are carried out with Matlab to verify the performance of the proposed synchronizing methods. The system parameters are chosen to be  $(a, b) = (10, 10)$  with which the systems yield chaos behaviors.

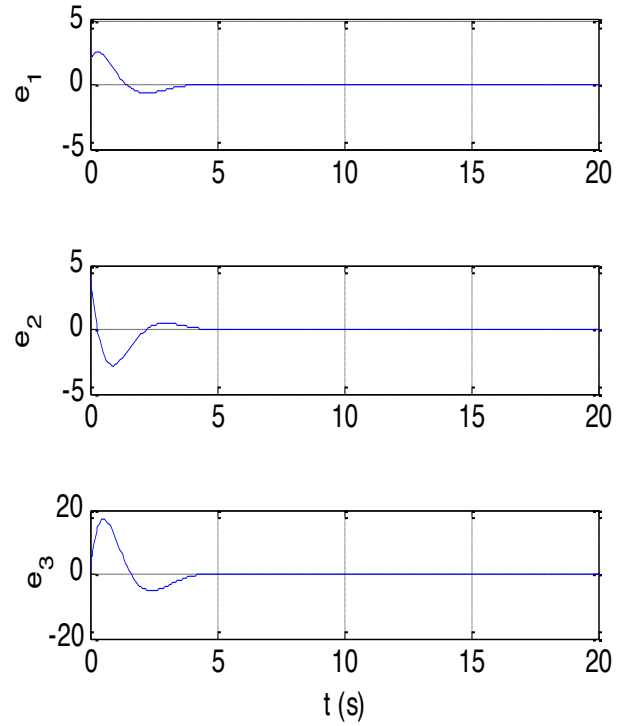
For the case of Theorem 1, the initial conditions of the drive and the response systems are chosen as:  $(x_1(0), x_2(0), x_3(0)) = (0, 1, 0)$  and  $(y_1(0), y_2(0), y_3(0)) = (2, 5, 2)$ , respectively.

The trajectories of the synchronization errors between (1) and (5) are presented in Fig. 3. It can be clearly observed that the synchronization of response and drive systems is achieved.

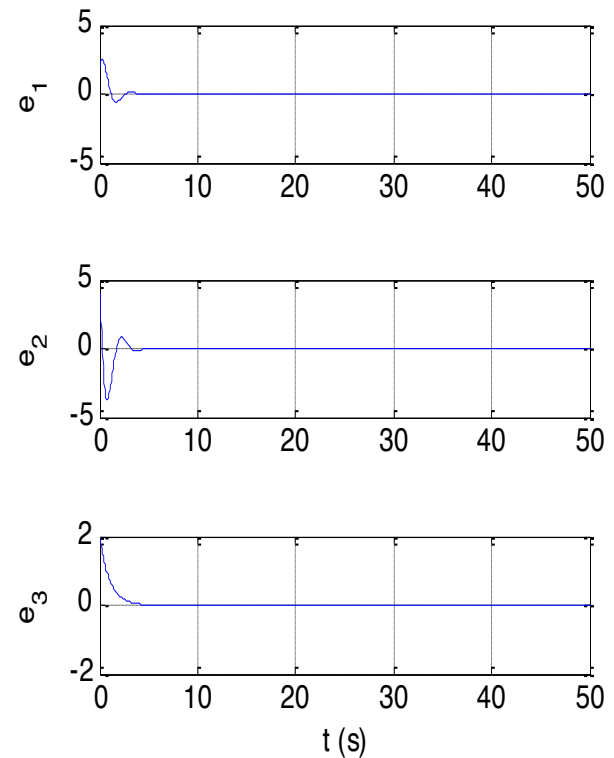
In the case of adaptive synchronizations, the initial value of unknown system parameter(s) of (1) and (16), (1) and (26) are given as  $\hat{b} = 9.0$  and  $(\hat{a}, \hat{b}) = (9.0, 10.2)$ , respectively. The trajectories of the synchronization errors with unknown parameter  $b$  and with unknown parameters  $a$  and  $b$  are shown in Fig. 4 and Fig. 5 respectively. It can be observed that the synchronization is achieved after a period of transient response and the estimated parameters approach some constants but not their true values which are presented in Fig. 6 and Fig. 7 respectively.



**Fig. 3.** Trajectories of synchronization error between the drive system (1) and the response system (5), in which the controller (13) is applied at  $t = 20$  s.



**Fig. 4.** Trajectories of synchronization error between the drive system (1) and the response system (15) with unknown parameter  $b$ , in which the controller (23) and the parameter update law (24) are applied at beginning.



**Fig. 5.** Trajectories of synchronization error between the drive system (1) and the response system (26) with unknown parameter  $a$  and  $b$ , in which the controller (28) and the parameter update law (29) are applied at beginning.

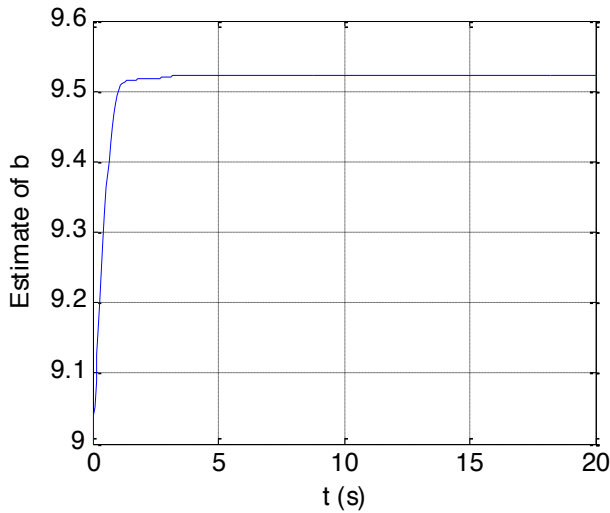


Fig. 6. Graph of  $\hat{b}$  when synchronizing systems (1) and (15).

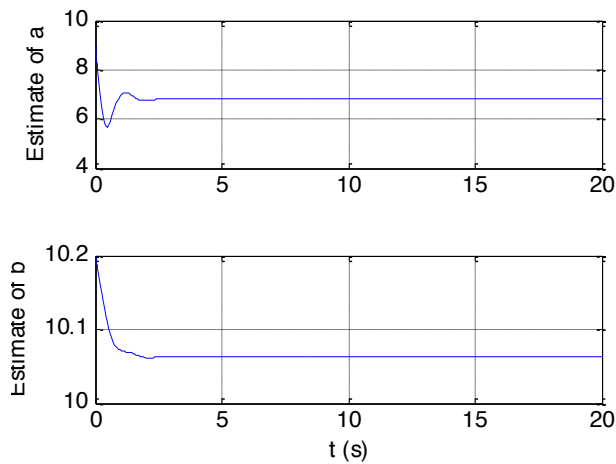


Fig. 7. Graph of  $\hat{a}$  and  $\hat{b}$  when synchronizing systems (1) and (26).

## 5. Conclusion

Memristor-based chaotic neural networks can reproduce the complex dynamics generated in the human brains associated with learning, associative memory, communication. Synchronization of the chaotic neurons plays a key role in the normal communication of the chaotic neural networks. This study addresses adaptive synchronization of novel chaotic neural systems which employ the HP memristor as a nonlinear part. The controllers and parameter update laws have been designed to adaptively synchronize the response systems with the drive system by using backstepping procedures. Simulation results demonstrate the effectiveness of the proposed approaches.

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