

Stability Analysis of Tunnel-Slope Coupling Based on Genetic Algorithm

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Abstract

Subjects in tunnels, being constrained by terrain and routes, entrances and exits to tunnels, usually stay in the terrain with slopes. Thus, it is necessary to carry out stability analysis by treating the tunnel slope as an entity. In this study, based on the Janbu slices method, a model for the calculation of the stability of the original slope-tunnel-bank slope is established. The genetic algorithm is used to implement calculation variables, safety coefficient expression and fitness function design. The stability of the original slope-tunnel-bank slope under different conditions is calculated, after utilizing the secondary development function of the mathematical tool MATLAB for programming. We found that the bearing capacity of the original slopes is reduced as the tunnels are excavated and the safety coefficient is gradually decreased as loads of the embankment construction increased. After the embankment was constructed, the safety coefficient was 1.38, which is larger than the 1.3 value specified by China's standards. Thus, the original slope-tunnel-bank slope would remain in a stable state.

Keywords: Genetic Algorithm, Tunnel Slope Coupling, Stability Analysis, Limit Equilibrium Method

1. Introduction

When the tunnel axis and the trend of slopes are the same, a covering layer of tunnels in a longer region is thin and the bias pressure tends to develop easily. Thus, it is essential to prevent this situation as much as possible. However, this situation appears inevitably in the practical engineering due to terrain selection. In this case, we need to analyze the tunnel-slope as a whole.

Currently, many methods are used to analyze the slope stability, such as the limit equilibrium method (LEM), the slip line method, the finite element method (FEM), the mathematical programming approach and the intelligent method, which were developed largely in domestic and foreign engineering practice. For instance, John et al. advantage of the composite algorithm for analysis of the slope stability, utilizing FEM to calculate the stress and strain of slopes and using LEM to carry out the stability analysis and adopting several examples of slopes for verification [1]. Zienkiewicz et al. applied the strength reduction technology to complete the slope stability analysis [2]. Now the numerical computation methods such as FEM and the finite difference method (FDM) are widely used. Meanwhile, Abdallah et al. successfully applied the Monte Carlo techniques to implement the slope stability analysis, searched the critical sliding surfaces of the slopes and used the Janbu method to calculate the stability coefficient of the slopes [3]. Additionally, Celestino et al. applied single-point directional migration linear programming method [4] and

Aria et al. adopted the conjugate gradient methods [5] in slope stability analysis. Meanwhile, Baker combined it with the Spencer method to confirm the non-arc critical sliding surface and the minimum safety coefficient [6].

Chinese scholars Gao et al. took into account Sandy Bay's large-section tunnel as a research background and applied ANSYS to analyze the stability of tunnels constructed by the double side drift method [7]. Zhang et al. utilized the improved discontinuous deformation analysis (DDA) to analyze the stability of a highway tunnel [8]. Based on the elastic-plastic theory and the non-associated flow rule, Ma et al. established the explicit calculation scheme of the double-shear uniform elastic-plastic constitutive models and analyzed the landslide scale and topographic features of a high slope before and after a landslide at the Southern Plateau of Jingyang County, in Shanxi Province of China [9]. In addition, Wang et al. conducted the slope stability analysis using fuzzy theory [10], and Feng et al. carried out limit equilibrium analysis of two groups of slopes that were made of parallel-jointed rocks, deduced the computational formula of the corresponding slope stability and compiled the analytical procedure of the slope stability based on the automatic search for potential sliding surfaces [11]. Furthermore, Wang et al. used FLAC^{3D} to analyze the stability of the slopes and tunnels under an unsymmetrical loading condition [12]. Likewise, Zhou et al. analyzed the slope deformation features caused by 3D deformation of a multiple-arch tunnel and slope with time, spatial distribution pattern and excavation procedures variation [13]. Moreover, Wang et al. performed the computational analysis of the tunnel-slope interaction models [14].

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To sum up, the LEM is an effective method for slope project design and stability analysis [15]. However, the combining analysis of the original slope, tunnel and the bank slope is lacking. In this study, we established a stability calculation model for combining the original slope, tunnel and bank slope using the Janbu slices method. Additionally, based on the GA, we would use the secondary development function of the mathematical tool MATLAB for programming and subsequently analyzed the stability of the combined slope, namely the original slope, the tunnel and the bank slope, under different working conditions.

2. Calculation Models

Slope failures at tunnel portals mainly involve the arc sliding surfaces. Thus, we undertook the typical cross section perpendicular to the tunnel axis as the research object and analyzed it by using a simplified Janbu method, since it is appropriate for the slope stability analysis of any sliding surface. Based on the results of such analysis, we hypothesized that the surface was a circular arc-shaped surface. The details of the model are presented in Figure 1.

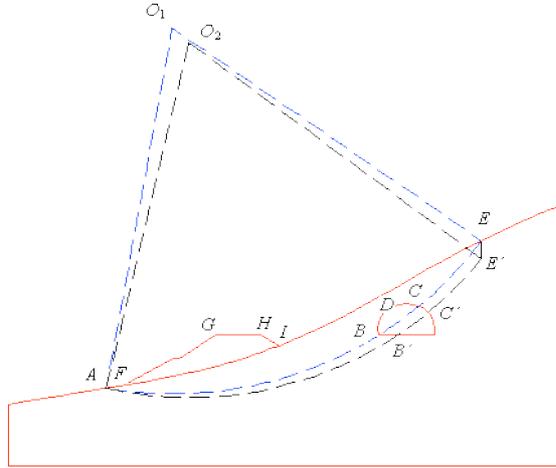


Fig.1. Limit equilibrium analysis model of the inlet section in the tunnel

While Figure 2 shows an analytical representation of the soil stripe of a potential sliding block, on which the acting force contains the weight of sub-blocks W_i , the horizontal component of the acting force among bars is W_i , the vertical component of the acting force among bars is Y_i , the shearing resistance (anti-skating force) at the bottom of the slices is T_i and the normal force at the bottom of the slices is N_i .

As the balance of the vertical force $\sum Y = 0$, we can obtain:

$$W_i - N_i \cos \alpha_i + Y_i - Y_{i+1} = 0 \quad (1)$$

Additionally, since the balance of the horizontal force $\sum X = 0$, we can obtain:

$$X_i - N_i \sin \alpha_i - T_i \cos \alpha_i - X_{i+1} = 0 \quad (2)$$

By applying the Mohr-Coulomb failure criterion, we obtained:

$$T_i = \frac{1}{F_s} (c_i l_i + N_i \cdot \tan \varphi_i) \quad (3)$$

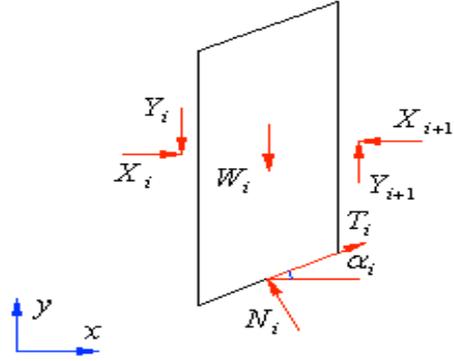


Fig.2. The potential sliding block used the simplified Janbu method

Combining equations (1), (2) and (3), we can get:

$$F_s = \frac{\sum \frac{1}{n_{ai}} \{c_i b_i + [W_i + (Y_i - Y_{i+1})] \cdot \tan \varphi_i\}}{\sum \{[W_i + (Y_i - Y_{i+1})] \cdot \tan \alpha_i\}} \quad (4)$$

Where $n_{ai} = \cos^2 \alpha_i (1 + \tan \alpha_i \cdot \tan \varphi_i / F_s)$.

According to the simplified Janbu method hypothesis, $Y_i - Y_{i+1} = 0$. Substituting, then equation (4) becomes:

$$F_s = \frac{\sum \frac{1}{n_{ai}} (c_i b_i + W_i \cdot \tan \varphi_i)}{\sum W_i \cdot \tan \alpha_i} \quad (5)$$

Where F_s stands for the safety coefficient, l_i stands for the length of the sliding surface ($l_i \approx b_i / \cos \alpha_i$) of the sub-blocks; b_i stands for the width of the rock-soil sub-block strips; α_i stands for the included angle between the sliding and the horizontal surfaces of the sub-block, c_i stands for the binding power on the sliding surface of the sliding sub-blocks, φ_i stands for the internal friction angle of rock and soil on the sliding surface, i stands for ordinal number of the analyzed slices and n stands for the number of sub-slices.

Thus, let:

$$A_i = c_i b_i + W \cdot \tan \varphi_i \quad (6)$$

$$B_i = W \cdot \tan \alpha_i \quad (7)$$

Then, Equation (5) becomes:

$$F_s = \frac{\sum \frac{1}{n_{ai}} A_i}{\sum B_i} \quad (8)$$

Since n_{ai} also contains F_s , we can assume that F_s equals 1 for trial calculation. If the calculated F_s does not equal 1, then we will solve the new n_{ai} and F_s by this equation. We next implement repeated iteration in this way

until two neighboring values of F_s largely approach each other. Usually, the requirements can be satisfied after 3-4 rounds of iterations and the iterations are generally convergent.

From Equation (8), we can obtain the safety coefficient of the slope stability under different working conditions. The factor is obtained for the assumed potential sliding surface. Usually, when the computational analysis is carried out the minimum safety coefficient and the corresponding potential sliding surface is being solved.

3. Design of GA and calculation of slope stability

3.1 Algorithm design

Generally, the traditional optimization methods include the dichotomization method, the pattern search method and the simple method. These methods are rapid and effective for finding the simple sliding surface on slopes. However, when the multi-stages or the number of soil layers is large and the difference in soil property is significant, the search tends to fall into the local minimum and cannot find the globally optimal solution. The GA can make up for the foregoing shortcomings [16], [17]. The calculation of the slope stability by the GA involves mainly three parts, namely the design variable, the safety coefficient expression and the fitness function design.

3.1.1 Design variables

A design for the central coordinate of a slip circle (X_0, Y_0) and the radius of the slip circle R (or the sliding depth $R - Y_0$) as the variables is shown in Figure 3. In the practical engineering, when the slope surfaces are out of shape simultaneously, the application of the traditional methods will make the solutions to left and right intersection points (l and r intersection points) between the arc slip surface and the slope more complex and difficult. There are studies in the literature [18] in which three X coordinates, namely the left and right intersection point between the arc slip surface and the slope and the center of circle are used as the design variables. In doing so, it is simple to confirm each geometrical parameter.

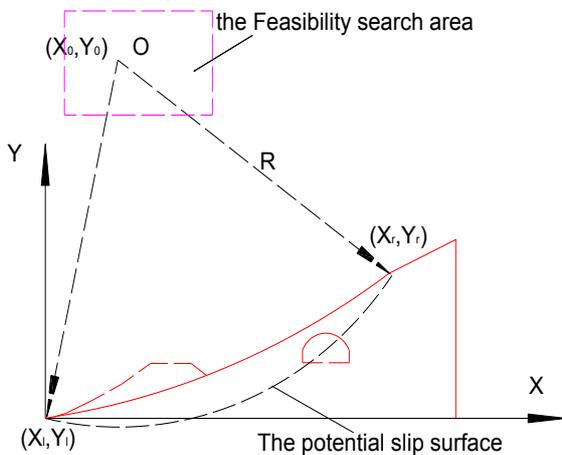


Fig.3. The slope stability of the genetic map variables design

To this end, X_l and X_r must first be confirmed, and then Y_l and Y_r are solved by the distribution function of the shapes of slope surfaces, explicitly the intersection points between the arcs slip surface and the slope surface. Lastly, according to the geometrical relationship of the potential sliding surfaces, as shown in Figure 4, the following computational formulas of Y_0 and R are elicited:

$$Y_0 = \frac{X_l - X_r}{Y_r - Y_l} \left(X_0 - \frac{X_l + X_r}{2} \right) + \frac{Y_l + Y_r}{2} \quad (9)$$

$$R = \sqrt{(X_l - X_0)^2 + (Y_l - Y_0)^2} \quad (10)$$

Now, since the vector composed of design variables, $X = [X_l, X_r, X_0]$, denotes the candidate solution to the optimization problem searching for the most dangerous sliding surface, then the feasible region of the solution is:

$$\Omega = \{X \mid X_l, X_r, X_0\} \quad (11)$$

Where Ω stands for the sub-region of three-dimensional European-style space R^3 .

3.1.2 Derivation of the safety coefficient expression

The coordinate system of the model for the calculation of the slope stability is shown in Figure 4. Take out, from the potential sliding slopes, a differential soil stripe with any unit thickness whose width is dx . All forces acting on the differential soil stripe are shown in Figure 1. Where y_1 is the functional equation of the slope lines and y_2 represents the functional equation of the potential sliding surfaces.

Thus, according to Figure 3, we can derive these equations:

$$dW_i \approx \gamma(y_1 - y_2)dx \quad (12)$$

$$\sin \alpha_i = \frac{x - x_0}{R} \quad (13)$$

$$\cos \alpha_i = \frac{\sqrt{R^2 - (x - x_0)^2}}{R} \quad (14)$$

$$\tan \alpha_i = \frac{x - x_0}{\sqrt{R^2 - (x - x_0)^2}} \quad (15)$$

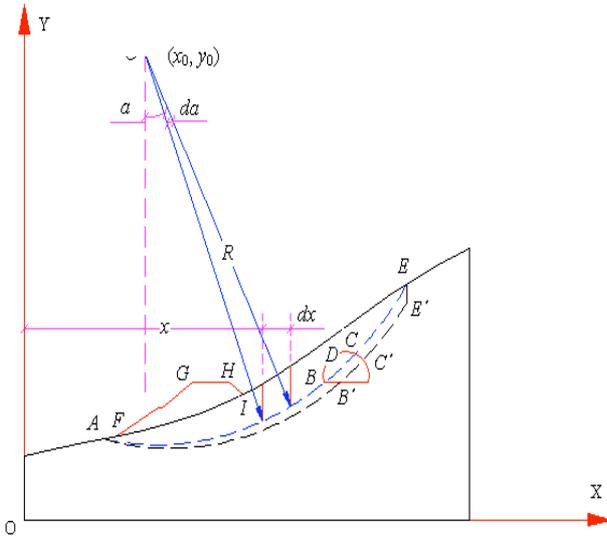


Fig. 4. Calculation of the slope stability of the integral expression of genetic

From the equations (12), (13), (14) and (15), the integral representation of the safety coefficient of the slope stability under different working conditions can be deduced.

3.1.3 Design of the fitness function

The objective function of the search for the most dangerous sliding surface of the slope is the safety coefficient of the slopes F_s . Each potential sliding surface that is confirmed by the design variable $X = [X_l, X_r, X_0]$ corresponds to the safety coefficient F_s . Thus, F_s is the function of X, i.e. $F_s = F_s(X)$. Now, since the problem involves the solutions to the minimum value of the target function and the fitness function, which should not be negative, the fitness function can be defined as:

$$F_s(X) = \begin{cases} 1/F_s(X) & F_s(X) > 0 \\ 0 & F_s(X) \leq 0 \end{cases} \quad (16)$$

The objective function is the expression of the calculation of the analytical models related to the slope stability at the entrances to tunnels during different construction stages.

Considering the search for the most dangerous sliding surface of the slope, we can treat it as the following optimization problem of the nonlinear function after establishing the design variables, inheritance coding, the design of target function and the fitness function and confirming the computing methods, hence:

$$\begin{cases} F_s = F_s(X) \\ X = [X_l, X_r, X_0]^T \\ X \in \Omega, \Omega \in R^3 \end{cases} \quad (17)$$

3.2 Algorithm implementation

The search for the most dangerous sliding surface of the slope is the optimization problem, which uses the safety coefficient F_s as the minimum objective function and uses the position of the sliding surface and the shape control parameter as state variables. In order to use the GA to solve the problem, it is essential to state the problem in a mode that is appropriate for using the GA to find solutions. In other words, the coding, the initial population, the fitness function and the genetic manipulation methods of the solutions to the problem should be individually confirmed [18], [19]. The specific procedures of the GA are shown as a flowchart in Figure 5. In this study, we executed the calculation of the slope stability based on the GA by applying the mathematical tool MATLAB secondary development technology.

4. Slope stability calculation

4.1 Parameter setting

The basic parameters needed for calculations are shown in the Table 1.

4.2 Results and analysis

The results were computed by operating the MATLAB program to calculate the potential sliding surface and the safety coefficient of the slopes under different working conditions. The evolutionary process of the calculation for each working condition is shown in Figure 6.

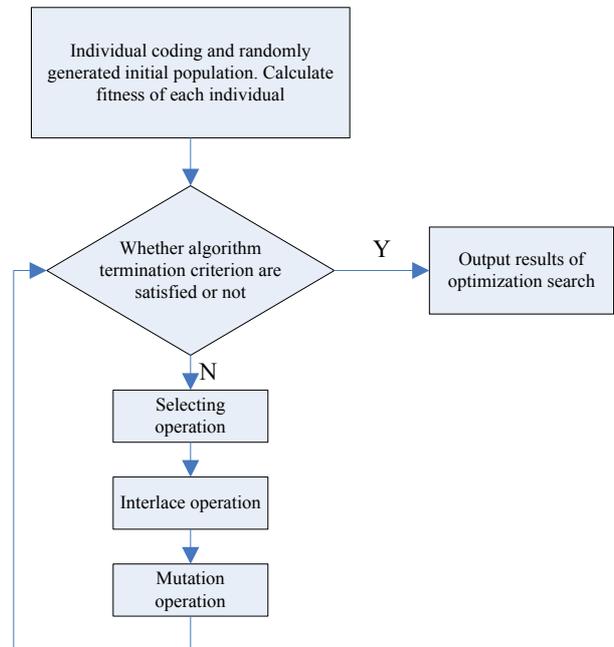


Fig. 5. Flow chart of steps of the basic genetic algorithm

Table 1. Parameters for calculating slope stability

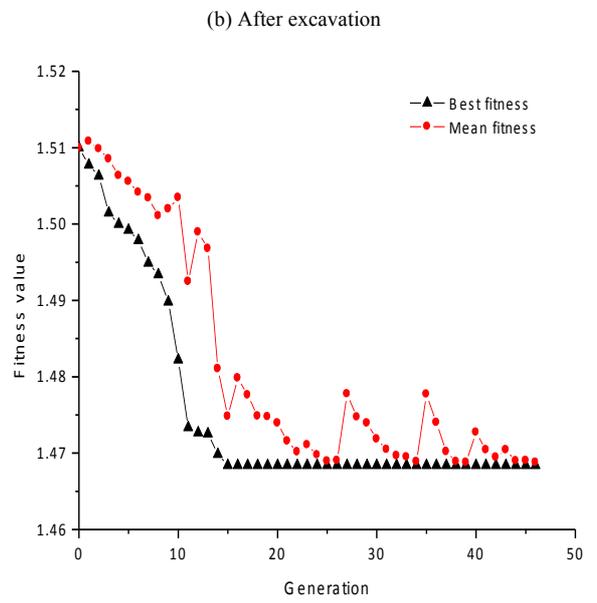
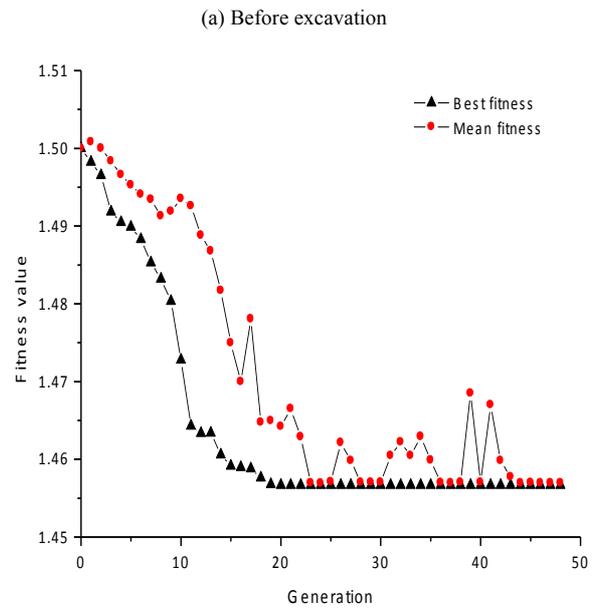
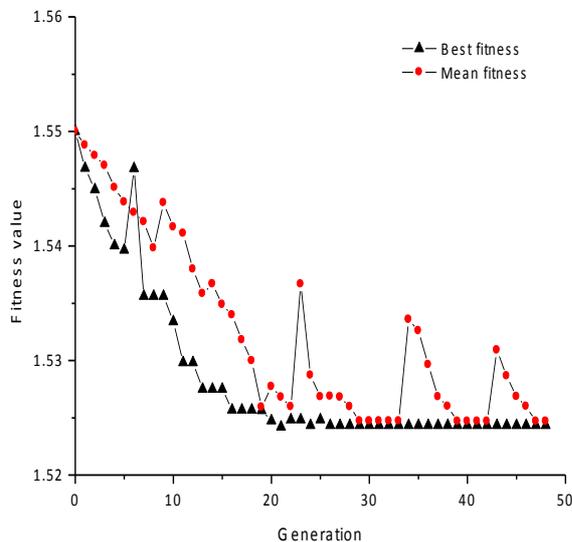
Formation lithology	Density (kN/m ³)	Frictional angle (°)	Cohesion (MPa)
Loess	1650	26.1	0.07
Zone of weathering	1900	28.0	0.29
Strong weathered rock formation	2100	30.7	0.94

In the chart of the genetic evolution procedure, the horizontal axis denotes the number of generations of the genetic evolution, the vertical coordinate denotes the fitness value. The red square points represent the average fitness value in each generation and the black triangle points stand for the best fitness value of each generation. The changes of fitness values in each generation produced by the algorithm are obvious during the initial generations of the genetic evolution.

As the evolution approaches an optimal solution, it gradually approximates to the optimal solution. Before the excavation of the tunnel, the optimal solution is approached when the calculation of the slope stability evolves to the 21st generation and stops at the 48th generation (Figure 6a). On the other hand, after the excavation of the tunnel, the optimal solution is approached when the calculation of the slope stability evolves to the 18th generation and stops at the 46th generation (Figure 6b).

Moreover, after the excavation and supporting of the tunnel, the optimal solution is approached when the calculation of the slope stability evolves to the 15th generation and stops at the 46th generation, and after the construction of the embankment, the optimal solution is approached when the calculation of the slope stability evolves to the 20th generation and stops at the 47th generation.

The Position parameters and the safety coefficient of the potential slip circles under the different working conditions, which were obtained through the calculations, are listed in the Table 2. The calculated results revealed that the bearing capacity of the original slopes decreases as the tunnels are excavated. In addition, as the loads of embankment construction increases, the safety coefficient of the slopes decreases gradually to 1.38, which is larger than 1.30 the value specified by standards. Therefore, the slopes tend to be ultimately stable.



(d) After construction of embankment

Fig.6. Slope stability based on the genetic algorithm evolutionary process map under different conditions

Table 2. The calculation results under different working conditions

Working condition	Position parameters of potential slip circles (m)			Minimum safety coefficient
	X	Y	R	
Before tunnel excavation	74.77	135.59	100.08	1.53
After tunnel excavation	71.31	139.64	105.24	1.46
After preliminary bracing of excavation	71.31	139.64	105.24	1.47
After construction of embankment	62.88	138.78	110.77	1.39

5. Conclusions

Based on Janbu slices method, the analytical model of the stability of the combined slope, namely the original slope-tunnel-bank slope was established.

GA was used to perform the calculations of the design variables, the safety coefficient expression and the fitness function. After applying the secondary development function of the mathematical tool MATLAB for programming, the original slope-tunnel-bank slope stability was calculated and analyzed under different conditions.

The bearing capacity of the original slope decreases as tunnels being excavated and the safety coefficient gradually decreases as the loads of embankment construction increasing. After construction of the embankment, the safety coefficient is 1.38, which is larger than the 1.3 value specified by China standards. Thus, the slopes tend to be fundamentally stable.

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