Effect of Double-line Parallel Shield Excavation on Adjacent Underground Pipelines

G. Wei, S. Hu, J.J. Xing and Q. Ye

1 Department of Civil Engineering, Zhejiang University City College, Hangzhou 310015, China
2 Institute of Geotechnical Engineering, Zhejiang University, Hangzhou 310058, China
3 Pypan Engineering Consultants Ltd., Hong Kong

Received 28 January 2016; Accepted 10 February 2016

Abstract

Using a formula for the total settlement of deep soil and the Winkler foundation model, the force model for underground pipelines is simplified to obtain the ultimate bending moments and to calculate the stress and strain due to double parallel shield tunneling construction. The effects of soil conditions, tunnel horizontal spacing L, pipeline material, pipeline depth h and soil loss ratio \( \eta \) on the force on an underground pipeline are investigated. The results show that changing the soil conditions and value of L can have a significant effect on the shape and magnitude of the pipeline’s ultimate moment curve, although changing the value of h has less of an effect. It is found that the larger the bending stiffness of the pipeline, the larger the distribution range of the ultimate bending moment and its peak value, but there is less of an effect on the shape of the curve. Changes in the ratio \( \frac{L}{\eta L} \) have some influence on both the shape of the curve and the values of the bending stiffness of the pipeline.

Keywords: Double-line parallel shield, Underground pipelines, Ultimate moment, Soil settlement

1. Introduction

In recent years, there has been an unprecedented development of subway construction. This mostly uses double-line horizontal parallel shield tunneling, which causes a greater amount of settlement and produces a wider settlement trough than single-line shield tunneling. The effects of double-line parallel shield tunneling on underground pipelines differ considerably from those of single-line shield tunneling. Therefore, an investigation of the effects of double-line parallel shield tunneling on adjacent pipelines is of great importance.

In this study, a modified two-dimensional (2D) Peck equation is used to calculate the vertical ground displacement at the pipe surface orthogonal to the direction of double-line horizontal parallel shield tunneling. Based on the Winkler foundation model, the ultimate bending moment, stress and strain on the pipeline due to tunneling are determined. Example calculations are performed to analyze the effects of tunneling on the pipeline.

The principal approaches to this problem have included numerical analysis and field measurements, but an analytical solution is lacking. Deformation of a pipeline due to single-line tunneling is generally represented by a symmetric normal distribution and is relatively simple. However, deformation due to double-line tunneling is complicated, being represented by an asymmetric distribution, since there are two conditions occurring, with “V” and “W” shaped distributions, respectively. Therefore, there is an urgent need for an analytical solution to evaluate the effect of double-line horizontal parallel tunneling on adjacent pipelines.

There have been numerous studies on the effect of single-line tunneling on pipelines. The principal methods employed have included analytical solutions and experimental studies. Wu et al. proposed an analytical solution for the displacement of a pipeline using the theory of beams on an elastic foundation, but their calculation was rather complex. Attewell et al. simplified the problem using the Winkler foundation model and provided analytical solutions. Vorster et al. provided a continuous elastic solution, the viability of which was validated by a centrifuge model experiment. However, the equation provided only an upper limit solution that was conservative, and the calculation was complex. Wang et al. analyzed the force on a pipeline orthogonal to the direction of tunnel excavation using the Logothetan equations on the basis of the Winkler foundation model in an attempt to remedy the deficiencies of the Attewell et al. solution. With regard to the deficiencies of the Wang et al. solution, Wei and Zhu proposed a theoretical approach based on the Winkler foundation model that allowed calculation of the ultimate bending moment and the deformation of a pipeline due to pipe jacking excavation. Zhang and Zhang proposed an equation describing the vertical displacement and internal force on a pipeline using the Pasternak foundation model, thus addressing the inherent drawbacks of the Winkler model. Taking into account the lack of continuity of pipeline connections, Zhang et al. proposed an analytical approach to the displacement and bending moment of a pipeline based on a modified Winkler foundation model. However, all of the studies mentioned here dealt with a single-line tunnel only, with no consideration of double-line horizontal parallel tunnels. To date, very few studies have been conducted on the effect of double-line tunneling on pipelines.
2. Methodology

2.1 Analysis of pipeline deformation and its mechanism of action

The effects on the ground foundation due to double-line horizontal parallel shield tunneling can be attributed to ground loss, friction at the shield, bulkhead additive thrust, grouting force, friction at the cutter head, etc., with ground loss being the primary factor. Because the range of impact of bulkhead additive thrust and friction at the shield is limited to the vicinity of the excavating face of the tunnel and because the induced ground surface displacement beyond a certain distance from the excavating face is basically zero, ground loss alone needs be considered in this study.

To simplify the calculation, only the force and displacement due to ground loss are evaluated in this study. An analysis shows that the force has the greatest effect when distance from the excavating face is basically zero, ground loss being the primary factor. Because the range of impact of ground loss, friction at the shield, bulkhead additive thrust, grouting force, friction at the cutter head, etc., with ground loss alone needs be considered in the present context [17].

The assumptions and simplification of the force model of the pipeline proposed by Wei [14] are adopted in this study, although the details will not be discussed here because of space limitations. To determine the ultimate bending moment of the pipeline, the ground surface settlement in the plane of the pipeline needs to be found.

Let the vertical displacement of the ground settlement curve due to double-line parallel shield tunneling be \( S(x) \). This can be viewed as the unloading process of the ground in the plane of the pipeline, and thus the force on the pipeline can be expressed as \( P = kS(x)d \), where \( k \) is the reaction force factor of the foundation in the plane of the pipeline, which can be further modified after experimental determination of the

Wei and Pang [18] evaluated the variation of the ground surface settlement curve due to double-line parallel shield tunneling and proposed that the shape of the curve is related to \( L \) and \( H + R \). Let \( C = L(H + R) \) be the relative horizontal distance factor of the tunnel: when \( C \leq 0.66 \), the ground surface settlement curve is “V” shaped (i.e., a normal distribution); when \( C > 1 \), the curve is “W” shaped; in the range \( 0.66 < C \leq 1 \), there is a transition from a “V” shaped to a “W” shaped curve.

The pipeline is closer to the tunnel than to the ground. Therefore, in this study, the relative horizontal distance factor of the pipeline is taken as \( G = L(H - h + R) \); when \( G \leq 0.66 \), the settlement curve of the pipeline is “V” shaped (a normal distribution) and when \( G > 1 \), the curve is “W” shaped, with a transition from a “V” shaped to a “W” shaped curve in the range \( 0.66 < G \leq 1 \).

For validation, we consider an example. Sun et al. [7] measured the settlement of a pipeline due to the construction of a double-line shield tunnel. The settlement of the pipeline appeared to be “V” shaped, but it did not match a normal distribution. The maximum settlement deviated to the side of the first tunnel. The method proposed in this study was used for validation, with parameters \( L = 13.2 \), \( H = 15.12 \), \( R = 3.17 \) and \( h = 0.9 \). A calculation gave \( G = 0.76 \), which is slightly larger than 0.66 and smaller than 1. Therefore, the settlement curve did not fit a normal distribution but was in the transition region from a “V” to a “W” shape, although closer to the former. The results of this validation calculation match favorably with the measured results, and the reliability of the proposed method is validated. The mechanism of the interaction between pipeline and ground due to double-line parallel shield tunneling is similar to that in the case of single-line shield tunneling, as explained by Wei [14].
reaction force factor of the foundation, \( k_0 \), from loading plate tests. The reaction force factor \( k_0 \) can be related to the stiffness and elastic characteristics of the foundation. Vesic [19] proposed the following expression for the reaction force factor of the foundation for a long beam (with \( b/b' < 10 \), where \( l \) and \( b \) are, respectively, the length and width of the beam):

\[
k' = \frac{0.66E_0}{b(1 - \mu^2)} \left( \frac{E_n b}{EI} \right)^{1/2}
\]

(1)

where \( E_0 \) is the modulus of deformation of the ground, \( \mu \) is Poisson’s ratio of the ground, \( E \) is the elastic modulus of the pipeline, \( EI \) is the bending stiffness of the pipeline and the width of the foundation beam, \( b \) is taken as equal to \( d \).

Taking account of the effect of the burial depth of the pipeline, Attewell et al. [9] proposed that the reaction force factor of the foundation in the plane of the pipeline is \( 2k' \), which is assumed in this study.

### 2.3 Calculation of ground surface settlement in the plane of the pipeline

Ground surface settlement in the plane of the pipeline due to double-line parallel shield tunneling can be precisely described by the amended 2D Peck equation proposed by Chen et al. [20]. The total ground surface settlement is taken to be equal to the sum of the ground surface settlements due to each tunnel. The equation for the settlement at any point \((x, z)\) of the ground is

\[
S_z(x) = \frac{S_{\text{max},f}}{(1 - z/H)^2} \exp \left[ \frac{-(x - 0.5L)^2}{2l_f^2 (1 - z/H)^2} \right] + \frac{S_{\text{max},f}}{(1 - z/H)^2} \exp \left[ \frac{-(x + 0.5L)^2}{2l_f^2 (1 - z/H)^2} \right]
\]

(2)

where \( S_{\text{max},f} = \pi R_i^2 \eta_f, S_{\text{max},f} = \pi R_i^2 \eta, S_{\text{max},f} = S_{\text{max},f} \) are the maximum ground surface settlements above the centerline due to the excavation of the tunnels, \( x \) is the horizontal distance to the central axis of the double-line shield, \( z \) is the vertical distance to the ground surface, \( \eta_f \) and \( \eta \) are the ground losses produced by each tunnel, \( l_f \) and \( l \) are the factors of the ground surface settlement troughs of each tunnel and \( n \) is the settlement trough parameter. When solving the above equation, a total of five parameters need to be determined: \( \eta_f, l_f, \eta, l \) and \( n \).

From a statistical analysis of the measured values of ground surface settlement due to tunneling in different geologic conditions, O’Reilly and New [21] proposed that the factor of the ground surface settlement trough \( i \) can be calculated from the empirical equation \( i = mH\), where \( m \) is a factor related to the earthiness. For sandy soil, \( m = 0.2–0.3 \); for clay soil and soft soil, \( m = 0.7 \); for medium clay soil, \( m = 0.5 \); and for hard clay soil, \( m = 0.4 \). The value of \( i_c \) can thereby be obtained [22]. The specific value of \( \eta_f \) has been found by Wei [23].

According to Chen et al. [20], \( i_f / i_f = 0.97, \eta / \eta_f = 0.96 \) and \( n \) should be in the range 0.35–0.85 for clay soil and 0.85–1.0 for sandy soil.

### 2.4 Solution for the force on the pipeline based on the Winkler model

As proposed by Wei and Zhu [14], when the stiffness of the pipeline is not large, the Winkler foundation model can be used to calculate the force on the pipeline. The differential equation describing the deformation of the pipeline due to tunneling is

\[
\frac{\partial^2 w}{\partial x^2} + 4\beta^2 w = 4\beta^2 S(x)
\]

(3)

where \( w \) is the vertical deflection of the pipeline and \( \beta = \sqrt{k_d / 4EI} \).

For an infinite beam, when a concentrated load \( P \) is applied at a point, the bending moment produced at a distance \( x \) from the loading point is

\[
M = \frac{P}{4\beta} \exp(-\beta x)(\cos \beta x - \sin \beta x)
\]

(4)

It is evident from the force on the pipeline equivalent that the infinitesimal concentrated load at a distance \( x \) from the center of the pipeline is \( dP = dk \cdot Sz(x) dx \).

Assume that the origin of coordinates is taken above the central axis of the double-line parallel tunnel, as shown in Fig. 1. It is also the vertical center of the pipeline. From the equations above, the ultimate bending moment at the vertical center of the pipeline is

\[
M_{\text{max}} = \int_0^\infty dM(x) = \int_{-\infty}^{-\infty} kS(x)dx \exp(-\beta x)
\]

(5)

\[(\cos \beta x - \sin \beta x)dx\]

The ground surface settlement in the plane of the pipeline can be calculated from Equation (2). The ultimate bending moment \( M_{\text{L}} \) at any point \((x, y, h)\) of the pipeline in the region affected by tunneling can be calculated as

\[
M_{\text{L}} = -EI \beta^2 \frac{2R^2 k_d}{i_f \sqrt{2\pi}} \left( 1 - \frac{h}{H} \right) \cdot \int_{-\infty}^{-\infty} \left[ \cos(\beta x + 0.5L - x_i) - \sin(\beta x + 0.5L - x_i) \right] \exp \left[ \frac{-(x - 0.5L)^2}{2l_f^2 (1 - h/H)^2} - \beta(x - 0.5L - x_i) \right] dx
\]

(6)

The ultimate stress on the pipeline can be calculated as

\[
\sigma = \frac{M_{\text{L}}}{W} = \frac{32M_{\text{L}}d}{\pi(d' - d^*)^3}
\]

(7)

where \( d' \) is the inner diameter of the pipeline. The ultimate strain produced on the pipeline can be calculated as

\[
\varepsilon = \frac{\sigma}{E} = \frac{32M_{\text{L}}d}{\pi E(d' - d^*)^3}
\]

(8)

If only the first tunnel is considered \((S_{\text{max},f} = 0)\), then the method discussed in this paper can be used for a single-line tunnel.
2.5 Assessment of the safety of the pipeline

It is proposed in this study that the pipeline is first affected by a single-line tunnel (i.e. the first tunnel) and then by the second tunnel. Therefore, to determine the safety of a pipeline affected by double-line parallel tunneling, two safety assessments should be conducted, first for a single-line tunnel (i.e. the first tunnel) and then for the double-line parallel tunnel.

When the stiffness of the pipeline is large, it can be assumed that no deformation occurs at the pipeline. Therefore, Equations (6), (7) and (8) can be used to calculate the ultimate bending moment, stress and strain on the pipeline, and safety can be evaluated in terms of the allowable stress and strain. When the stiffness of the pipeline is small, it can be assumed that the settlement of the pipeline is equal to the ground surface settlement in the plane of the pipeline due to double-line shield tunneling. Equation (2) is used for this calculation. Once the settlement curve of the pipeline has been obtained, the maximum angle of rotation of the pipeline can be calculated and the safety evaluated in terms of the allowed angle of rotation.

The method proposed in this study is conservative. It is assumed that the pipeline is orthogonal to the direction of the double-line parallel tunnel, that the ground is homogeneous (layering of the ground is not considered) and that only ground loss is significant.

3. Results

3.1 Analysis of example calculations

The following conditions are assumed. This is a double-line horizontal parallel shield tunneling project with an excavating radius $R = 3.17$ m, the burial depth of the tunnel centerline $H = 12$ m, the horizontal distance between the centerlines of the two tunnels $L = 12$ m and the right tunnel is excavated first. The tunnels are excavated in clay soil with a Poisson’s ratio of 0.35, a modulus of deformation $E_0 = 3087.6$ kPa and a unit weight of 18 kN/m$^3$. Assuming that $\eta_f = 1.56$% owing to the excavation of the first tunnel, the width of the ground surface settlement trough can be calculated using the approach of O’Reilly and New [21] as $l_f = 6$ m. According to Chen et al. [20], $i_f/\eta_f = 0.96$, and it is clear that $\eta_f = 1.50$%. If $i_f/\eta_f = 0.97$, then $i_f = 5.8$ m. A value of $n = 0.4$ is taken.

There is an underground rigid steel pipeline orthogonal to the direction of excavation of the tunnel, lying above the tunnel. The outer radius of the pipeline is 0.4 m, the centerline is a depth $h = 1.5$ m below the ground surface, the tube thickness is 16 mm, $E = 2.06 \times 10^5$ MPa and the bending stiffness $EI = 1246300$ kN m$^2$. The reaction force of the foundation is calculated from Equation (1) and doubled, so that $k = 3219$ kN/m. It is calculated that $\beta = 0.151$.

3.2 Effect of geologic conditions

To evaluate the effect of variation of geologic conditions on the pipeline with other conditions held constant, it is assumed that the tunnel is excavated in round grain sand with a Poisson’s ratio of 0.25, a modulus of deformation of 14 320 kPa and a unit weight of 18 kN/m$^3$. Using O’Reilly and New’s [21] approach, it is calculated that $i_f = 3.6$ m.

Following the recommendation of Chen et al. [20] that $i_f/\eta_f = 0.97$, it can be calculated that $i_f = 3.49$ m, and, taking $n=0.85$, the ground loss rate is identical to that with clay soil. The reaction force factor of the sandy soil foundation is calculated from Equation (1) and doubled, so $k = 15.881 \times 10^5$ kN m$^2$, and thus $\beta = 0.225$.

The ultimate bending moment of the pipeline due to double-line shield tunneling calculated using the method proposed here is shown in Fig. 4, from which it can be seen that geologic conditions significantly affect the shape and magnitude of the ultimate bending moment curve of the pipeline. There are three negative and two positive peak values of the bending moment for a rigid pipeline in sandy soil, with the pipeline being subjected to a maximum ultimate negative bending moment $M_{\text{max}} = -813.66$ kN m at $x = 16$ m and a maximum ultimate positive bending moment $M_{\text{max}} = 1426.76$ kN m at $x = 6$ m. $M_{\text{max}}$ is much larger than $M_{\text{max}}$. Because of the interaction between the two tunnels, the curve is slightly asymmetric. For clay soil, the bending moment curve of the rigid pipeline is close to a normal distribution. There are two negative and one positive peak values of the
bending moment. $M'_{\max}$ is also larger than $M'_{\max}$, with $M'_{\max} = 258.31 \text{ kN} \cdot \text{m}$ and $M'_{\max} = 352.89 \text{ kN} \cdot \text{m}$.

If the double-line tunnel is excavated in clay soil, $M_{\max}^+ = 352.89 \text{ kN} \cdot \text{m}$, the ultimate tensile stress is 90.48 MPa and the ultimate tensile strain is $4.39 \times 10^{-4}$. If it is excavated in sandy soil, $M_{\max}^- = 1426.76 \text{ kN} \cdot \text{m}$, the ultimate tensile stress is 365.81 MPa and the ultimate tensile strain is $1.78 \times 10^{-3}$.

According to Wu [24], the allowed stress for a steel pipe $[\sigma] = 166.3 \text{ MPa}$. Therefore, the rigid pipeline is safe in clay soil, but in sandy soil, both single- and double-line tunneling result in the allowed stress being exceeded and both are therefore dangerous.

### 3.3 Effect of horizontal distance $L$ between double-line tunnels

To evaluate the effect of horizontal distance, the value of $L$ is changed, while the other parameters are identical to the standard conditions. The ultimate bending moment of a pipeline at different values of $L$ calculated using the method proposed here is shown in Fig. 6. It can be seen that varying $L$ has a significant effect on the shape of the ultimate bending moment curve of the pipeline and the value of the bending moment at the central axis. First, when $L = 8 \text{ m}$, the behavior of the ultimate bending moment of the pipeline is similar to that in the case of single-shield tunneling, with a maximum positive bending moment at the central axis and $M_{\max}^+ = 669.74 \text{ kN} \cdot \text{m}$, which is larger than the value 494.8 kN·m due to the excavation of the first tunnel. This suggests that when the horizontal distance between the double-line shields is small, the detrimental effect on the pipeline is greater than in the case of single-line shield tunneling. Second, with increasing $L$, the ultimate bending moment of the pipeline varies significantly, in particular at the central axis, where the positive bending moment changes toward a negative bending moment. When $L = 20 \text{ m}$, at the central axis, $M_{\max}^- = 216.22 \text{ kN} \cdot \text{m}$, and the curve around the central axis transforms from a “V” shape to a “W” shape. If the first (single-line) tunnel is excavated in clay soil, $M_{\max}^- = 335.33 \text{ kN} \cdot \text{m}$, which occurs at $x = -16 \text{ m}$; when $L = 20 \text{ m}$, $M_{\max}^- = -210.65 \text{ kN} \cdot \text{m}$, which occurs at $x = -24 \text{ m}$.
The value of the burial depth \( h \) of the centerline of the pipeline is varied from 1, 2, 3 to 4 m, with other parameters being identical to the standard conditions. The ultimate bending moment of the pipeline at different values of \( h \) calculated using the method proposed here is shown in Fig. 7. It can be seen that the variation of \( h \) has little effect on the ultimate bending moment of the pipeline, and that the shapes of the four curves are similar. Under the same conditions, the higher the value of \( h \) (i.e., the closer the pipeline is to the tunnel), the larger is the induced ultimate bending moment. When \( h = 1 \) m, \( M_{\text{max}}^1 = 355.56 \) kN·m, which occurs at \( x = -2 \) m; when \( h = 4 \) m, \( M_{\text{max}}^1 = 371.76 \) kN·m (an increase of 4.56%), which occurs at \( x = -4 \) m. The maximum negative bending moment occurs around \(-18 \) m in all cases. When \( h = 1 \) m, \( M_{\text{max}}^1 = -252.52 \) kN·m, and when \( h = 4 \) m, \( M_{\text{max}}^1 = -290.47 \) kN·m (an increase of 15.03%).

3.5 Effect of the ratio \( \eta_l / \eta_f \)

The interaction between the two tunnels is a significant factor. Different ratios \( \eta_l / \eta_f \) are examined in this study. It is assumed that \( \eta_f \) due to the first tunnel is 1.56%, and \( \eta_l / \eta_f \) is taken as 0.8, 1.0 and 1.2, giving \( \eta_l = 1.25\% \), 1.56% and 1.87%, respectively. The effect of the interaction of the two tunnels on the pipeline is evaluated, with other parameters identical to the standard conditions.

3.6 Effect of pipeline material

In this part of the study, the pipeline material is varied, as are the material parameters and the thickness of the pipeline tube. For pipelines affected by double-line parallel tunneling, two safety assessments should be made, first for a single-line tunnel (i.e., the first tunnel), then for a double-line parallel tunnel.

The ultimate bending moment of the pipeline calculated using the method proposed here is shown in Fig. 9 for a number of different materials. It can be seen that the pipeline material has a significant effect on the magnitude of the ultimate bending moment, but not on the shape of the curve. The distribution range and the peak value are largest for a steel pipeline and smallest for a PVC pipeline. The ultimate bending moment curves for cast iron and concrete pipelines are very similar, although the magnitude for the concrete pipeline is slightly larger. These results suggest that pipelines with a larger bending stiffness have a larger distribution range and peak value of the ultimate bending moment.

### Table 1. Parameters of the pipeline

<table>
<thead>
<tr>
<th>Pipe material</th>
<th>Tube thickness (mm)</th>
<th>Elastic modulus (MPa)</th>
<th>Bending stiffness ((\text{kN} \cdot \text{m}^2))</th>
<th>Poisson's ratio</th>
<th>k ((\text{kN/m}))</th>
<th>(\beta)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Steel</td>
<td>16</td>
<td>2.06×10^3</td>
<td>1,246,300</td>
<td>0.3</td>
<td>3219</td>
<td>0.15</td>
</tr>
<tr>
<td>Cast iron</td>
<td>12</td>
<td>9.0×10^4</td>
<td>414,900</td>
<td>0.27</td>
<td>5</td>
<td>0.20</td>
</tr>
<tr>
<td>Concrete</td>
<td>60</td>
<td>2.5×10^4</td>
<td>480,000</td>
<td>0.17</td>
<td>3486</td>
<td>0.19</td>
</tr>
<tr>
<td>PVC</td>
<td>30</td>
<td>2.26×10^3</td>
<td>24,871</td>
<td>0.35</td>
<td>4461</td>
<td>0.43</td>
</tr>
</tbody>
</table>

4. Conclusion

For pipelines affected by double-line parallel tunneling, two safety assessments should be made, first for a single-line tunnel (i.e., the first tunnel), then for a double-line parallel tunnel.

With increasing \( L \), the distribution curve of the ultimate bending moment of the pipeline varies significantly. In particular, at the central axis, where the positive bending
moment changes toward a negative bending moment, the shape of the curve transforms from a “V” to a “W” shape, and the location of the maximum negative bending moment of the pipeline drifts toward the two sides and its value decreases.

The ratio \( \eta_i / \eta_j \) affects the shape of the curve and the magnitude of the ultimate bending moment of the pipeline. With increasing \( \eta_i / \eta_j \), the ultimate bending moment of the pipeline increases gradually, and the location of the peak positive bending moment shifts.

Geologic conditions have a significant effect on both the shape and magnitude of the ultimate bending moment curve of the pipeline. The pipeline material has a significant effect on the magnitude of the ultimate bending moment, but not on the shape of the curve. For pipelines with a larger bending stiffness, the distribution range and peak value of the ultimate bending moment are larger. The value of \( h \) has little effect on the ultimate bending moment of the pipeline.

Acknowledgements
This work was financially supported by public technology projects of Science Technology Department of Zhejiang Province (NO:2016C33051) and science and technology project plan of Ministry of Housing and Urban-Rural Development of the People's Republic of China in 2015 (NO:2015-KS-026).

References
2. Tadeusz Majcherczyk, Zbigniew Niedbalski, Michał Kowalski., “3D numerical modeling of road tunnel stability—the Laliki project”. Archives of Mining Sciences, 57(1), 2012, pp.61-78.