Journal of Engineering Science and Technology Review 9 (1) (2016) 182-186

JOURNAL OF
Engineering Science and Technology Review

www.jestr.org

Research Article

Probability Researches of Design Basis Flood Level in Shidao Bay

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Received 11 December 2014; Accepted 15 February 2015

Abstract

Design Basis Flood Level(DBFL) is one of the important parameters in nuclear power plant siting in coastal area. This paper proposes a new model of Instantaneous Sea Surface Height(ISSH) to calculate DBFL. The model is validated and compared with bivariate extreme distribution model and independent variables model, using observed series from Shidao bay in Shandong province. Results show that the model is more suitable for representing the DBFL. The joint probability model gets greater return period and smaller return level than ISSH model, because it adopts an association parameter derived from the linear correlation coefficient. Independent variables model is the most reliable in safety but wasteful in economy.

Keywords: Instantaneous sea surface height; Bivariate extreme distribution; Gumbel distribution; Return level; Return period

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1. Introduction

Ocean observation tends to separate elements for records, such as surge level, tide level, wave height and so on. Indeed, design Basis Flood(DBF) can be described as a multivariate event whose main components are water level and wave height, which are mutually correlated. However, water level and wave height had been considered as independent variables for calculating Design Basis Flood Level(DBFL). Annual extreme values of water level and wave height were separately recorded, however, in fact, they do not occur at the same time in statistics. This kind of analysis provided a limited and conservative assessment for DBFL. Since 1980s, extensive researches develop marine engineering design criteria based on the joint probability distribution, pointing out the importance of correlation factor. Theoretically, two bivariate extreme value distributions with Gumbel marginals, namely Gumbel mixed model and Gumbel logistic model, have been studied by Gumbel and Mustafi[1], and Oliveria[2]. They have been largely applied to correlated hydrological extreme events by Yue et al.[3, 4], Dong et al.[5,6], Xie et al. [7], Yu et al. [8], and Stuart et al.[9]. Lately, various types of bivariate copula function models develop quickly and have been applied in various situations by Dong et al.[10], Wang[11], Fan et al.[12], Helene et al.[13], and Kong et al[14]. But the limitation of these models is that the association parameter is derived from the linear correlation coefficient.

Based on the aforementioned studies, this paper proposes a model of Instantaneous Sea Surface Height(ISSH). ISSH is the height (or topography or relief) of the ocean's surface, which is most obviously affected by the tidal level, surge level, rainfall, wind-generated waves and so on. ISSH itself already contains the mutual correlations of various elements that can influence the height. Furthermore, one-dimensional extreme value distribution could be applied for ISSH data so that it is possible to obtain various return levels of DBFL given the return periods, and vice versa. Unfortunately, it is difficult to collect ISSH series in historical statistics because of the separate observations. However, extreme series such as maximum wave height and the corresponding water level, maximum water level and the corresponding wave height can be obtained. Probably, an idea is proposed, that takes the larger one of two ISSH converted from aforementioned statistics each year, as the substitute of actual ISSH series. Section 2 presents three models in a mathematical way, and details the bivariate extreme value distribution theory adopted in this paper. Section 3 presents a practical application of the three models, and the results are given at last. Section 4 concludes the superiority of the ISSH model for representing DBFL compared with another two models.

2. Theoretical Part

2.1. Mathematical Models

Hypothetically, the ISSH distribution function of the time variable t is $f(t)$, the water level $h(t)$, and the wave height $\xi(t)$. Also, $\langle \bullet \rangle$ is defined for obtaining the return levels of DBFL as return periods of $\bullet$ series are given. Thus, ISSH can be expressed as follows:

$$f(t) = h(t) + \xi(t). \tag{1}$$

After derivation for these functions, on the assumption, ISSH gets the annual extreme value at t_1 time series, water level t_2, and wave height t_3. Generally, t_2 is not equal to t_3.

In independent variables model, h and ξ will be considered as independent variables. The algorithm form for calculating DBFL can be expressed as follows:

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ISSN: 1791-2377

$$H = \langle h(t_2) + \xi(t_3) \rangle, \tag{2}$$

where it apparently results in largest return level given the return period. Therefore, this model is the most reliable in safety but wasteful in economy.

Since 1980s, joint probability distribution model is put forward, which considers the temporal relevance between h and ξ series. This model is applied to $\{h(t_2), \xi(t_2)\}$ and $\{h(t_3), \xi(t_3)\}$ series. Given the same return period, the larger of two return levels is chosen as the final result. The algorithm form for calculating DBFL can be expressed as follows:

$$H = \max\{\langle h(t_2), \xi(t_2) \rangle, \langle h(t_3), \xi(t_3) \rangle\} \tag{3}$$

In some extent, this model solves the temporal relevant problem between water and wave. But the limitation is that the association parameter is derived from the linear correlation coefficient. Therefore, it might still be different from actual interaction.

ISSH contains the actual interaction of water and wave. In practical statistics, $h(t_2)$、$\xi(t_2)$、$h(t_3)$ and $\xi(t_3)$ series could be found, except $f(t_1)$ which is undoubtedly actual ISSH series. Thus, the larger one of two ISSH series in each year is chosen as the substitute of the actual ISSH series. The algorithm form for calculating DBFL can be expressed as follows:

$$H = \langle \max\{f(t_2), f(t_3)\} \rangle = \langle \max\{h(t_2) + \xi(t_2), h(t_3) + \xi(t_3)\} \rangle \tag{4}$$

2.2. The Gumbel Logistic Model

The Gumbel logistic model with standard Gumbel marginal distribution was originally proposed by Gumbel[1,2] as follows:

$$F_{XY}(x,y) = \exp\{-[(-\ln F(x))^{\frac{1}{\alpha}} + (-\ln F(y))^{\frac{1}{\alpha}}]^{\alpha}\} \quad (0 \le \alpha \le 1) \tag{5}$$

where $F(x)$ and $F(y)$ are, respectively, the marginal distribution functions of random variables X and Y, and are expressed as

$$\begin{cases} F_X(x) = \exp[-\exp(-x)], \\ F_Y(y) = \exp[-\exp(-y)] \end{cases} \tag{6}$$

and where α $(0 \le \alpha \le 1)$ is the parameter describing the association between the two random variables X and Y. The estimator of α is given by[2]

$$\alpha = \sqrt{1 - \rho_{XY}} \quad (0 \le \rho_{XY} \le 1), \tag{7}$$

where ρ_{XY} is the product-moment correlation coefficient estimated by

$$\rho_{XY} = \frac{E[(X - \mu_x)(Y - \mu_y)]}{\sigma_x \sigma_y}, \tag{8}$$

where (μ_x, σ_x) and (μ_y, σ_y) are, respectively, the means and standard deviations of X and Y.

Where $\alpha = 1$, the bivariate distribution splits into the product of the two marginal distributions, and becomes

$$F(x,y) = F(x)F(y). \tag{9}$$

This means the independent case and the product-moment correlation coefficient is equal to zero.

The joint return periods $T_{xy}(x,y)$ exceeding certain values of variables X and Y associated with the event ($x > X$ and $y > Y$, i.e., both of the two values are exceeded) , can be represented by

$$T_{xy} = \frac{1}{P(X > x, Y > y)} = \frac{1}{1 - F_x(X \le x) - F_y(Y \le y) + F(X \le x, Y \le y)} \tag{10}$$

3. Application

3.1. Sample Definition

20-year consecutive annual extreme data from 1985 to 2004, which were observed from Shidao bay in Shandong province, were made use of to demonstrate the applicability of ISSH model. Due to the limitation of statistics, ignoring the tidal level and rain-offs, the surge level and wave height were just under consideration in this paper. The ISSH height can be expressed as surge level plus wave height(one percent cumulative wave height multiply by 0.6, because of the asymmetry of waves). Four samples converted from the existing statistics can be detailed as follows:

1） Sample 1 (S1) : maximum surge level and the corresponding wave height ;
2） Sample 2 (S2) : maximum wave height and the corresponding surge level ;
3） Sample 3 (S3) : ISSH, the larger one of two ISSH converted from S1 and S2 each year;
4） Sample 4 (S4) : ISSH in the case where surge level and wave height random variables are considered as the independent random variables. Maximum surge level plus maximum wave height each year can be considered as an independent theoretical ISSH value.

3.2. Model Analysis

This paper selects Gumbel distribution, extreme value distribution I (EV1), widely used in engineering domain of the world[15,16], as the single-variable analysis model for sample S3 and S4. Generally, the location and scale parameters of the Gumbel distribution can be estimated by maximum likelihood method (ML) or the method of moments (MM). However, an idea of identifying all the model's parameters completely through marginal distributions of the variables via the MM, is proposed by Gumbel and Mustafi[1]. And the reliability for estimating the association parameter estimation using ML method remains uncertain. In consideration of the uncertainty and in order to keep the consistency of the methodology, this paper completely identify all the models' parameters via the MM. The efficiency of this estimation method, has been tested by Shi[17]. The location and scale parameters could be estimated by the MM method as follows:

$$\begin{cases} \sigma = \dfrac{\sqrt{6}}{\pi} S \\ \mu = \overline{X} - 0.5776\sigma \end{cases}, \tag{11}$$

where $\overline{X}$ and S are the mean and standard deviation of the sample series, respectively. The estimated parameters of all the samples are listed in Table 1.

Table 1. Estimated parameters of Sample S1, S2, S3 and S4

Sample Number	Marginal Variable	Location Parameter	Scale Parameter	Association Parameter
S1	Surge Level Wave Height	0.5318 1.7219	0.0896 0.8043	0.8931
S2	Surge Level Wave Height	0.0605 4.4677	0.1057 0.3996	0.7209
S3	—	4.5544	0.4598	—
S4	—	5.0326	0.4319	—

Guo[18] (1990) have demonstrated that the Gringorten formula is unbiased for EV1 quantile estimation. The non-exceedance probability can be estimated by the Gringorten formula:

$$P_k = \frac{k-0.44}{N+0.12} \quad \text{or}$$

$$P_{ij} = P(X \le x_i, Y \le y_j) = \frac{\sum_{m=1}^{i}\sum_{l=1}^{j} n_{ml} - 0.44}{N+0.12}, \tag{12}$$

where N is the total number of observations ($N = 20$), and P_k or P_{ij} is the cumulative frequency, the probability that a given value is less than the kth or n_{ml}th smallest observation in the total N observations. Figs.1 (a), (b), (c) and (d) show the fit of the marginal Gumbel distributions of surge level and wave height series in sample 1 and 2, on the Gumbel paper. The empirical and theoretical joint probabilities for all the samples are illustrated in Figs. 2 (a), (b), (c) and (d), respectively.

The Kolmogorov-Smirnov test is executed to test the goodness of fit of the models. The tested results show that all the distributions adopted by samples are accepted at the significant level 0.05. Thus, it is concluded that all the models are respectively suitable for all the samples.

3.3. Application

Given an occurrence probability or a return period of the DBFL, various occurrence combinations of surge level and wave height can be obtained. On the one hand, these various scenarios are especially useful for hydrological engineering design and management. On the other hand, the maximum likelihood combination must be determined sometimes. In this paper, tangent line method, which has been used by Liu, Wen and Wang[19], is chosen for the calculation of the most reasonable combination of surge level and wave height. Given return periods=[50, 100, 200], the corresponding contours and tangent lines of the joint periods of Sample S1 and S2 are plotted in Fig.3. The tangent points means the maximum likelihood combination of surge level and wave height. Furthermore, the DBFL is equal to the value of surge level plus wave height.

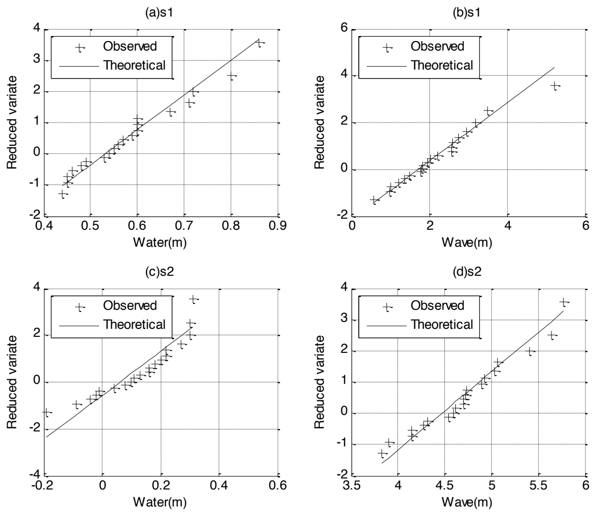

Fig. 1. (a) Distribution of surge level series of Sample 1, (b) distribution of wave height series of Sample 1, (c) distribution of surge level series of Sample 2, (d) distribution of wave height series of Sample 2.

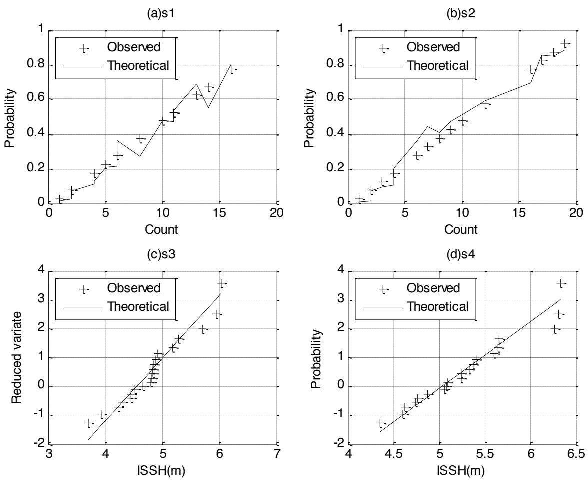

Fig. 2. (a) Joint distribution of Sample 1, (b) joint distribution of Sample 2, (c) distribution of ISSH of Sample 3, (d) distribution of ISSH of Sample 4.

Given the same return periods, the return ISSH levels of Sample S3 and S4 are also derived from the ISSH model and independent variables model. For the purpose of comparison, the return ISSH levels of Sample S1, S2, S3 and S4 are listed in Tab.2.

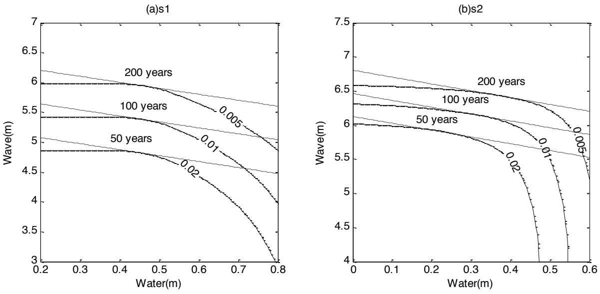

Fig. 3. (a) Tangent graph of Sample S1 corresponding to given return periods=[50, 100, 200], (b) Tangent graph of Sample S2 corresponding to given return periods=[50, 100, 200].

Table 2. Comparison for return ISSH levels of Sample S1, S2, S3 and S4

Return Period (years)	ISSH (m)			
	S1	S2	S3	S4
50	5.28	6.13	6.35	6.72
100	5.84	6.46	6.67	7.02
200	6.40	6.80	6.99	7.32

Similarly, the return periods corresponding to given return ISSH levels=[6.35, 6.67, 6.99], can be obtained by the tangent method, and are shown in Fig.4. All the return periods of Sample S1, S2, S3 and S4 are presented for comparison in the Tab.3.

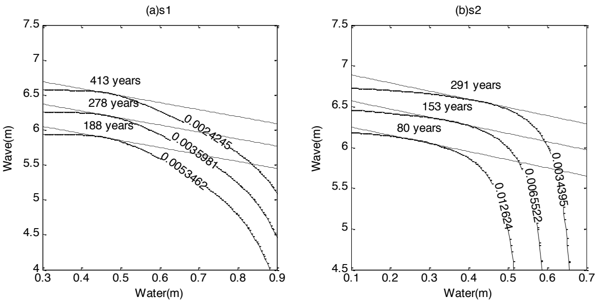

Fig. 4. (a) Tangent graph of Sample S1 corresponding to given return ISSH levels=[6.35, 6.67, 6.99], (b) Tangent graph of Sample S2 corresponding to given return ISSH levels=[6.35, 6.67, 6.99].

Table 3. Comparison for return periods of Sample S1, S2, S3 and S4

ISSH (m)	Return Period (years)			
	S1	S2	S3	S4
6.35	188	80	50	22
6.67	278	153	100	45
6.99	413	291	200	94

3.4. Results

In a condition of the same return period, the return ISSH level of Sample S3 is greater than S2 about 0.2m, and S4 is greater than S3 about 0.4m, and S1 is the smallest. Moreover, in a condition of the same return ISSH level, the return period of Sample S3 is less than S1 and S2, and S4 is the smallest.

4. Conclusions

This study presents a model for the direct use of ISSH to analyze the DBFL and the results are compared with the methodologies where surge level and wave height are considered as two correlated or independent extreme random variables that are represented by the Gumbel distribution. Indeed, this approach contains the real complicated correlation between wave height and water level variables, and the association parameter is not appropriate to be simply derived from the linear correlation coefficient. The results point out that the ISSH model provides additional information which cannot be obtained by other two models.

Results show that the ISSH model is more suitable for representing the DBFL; The joint probability model gets greater return period and smaller return level than ISSH model, because it adopts an association parameter simply derived from the linear correlation coefficient; Independent variables model is the most reliable in safety, but the over-estimation will lead to increased cost largely in hydrological engineering planning, design, and management. The proposed ISSH model are made to provide information which allows to ensure safety of people or relevant structures, systems and components, e.g. of critical infrastructure. Therefore, it is urgent for us to add ISSH observations to the conventional observations offshore

Acknowledgements
This work was financially supported by the Researches of Anti-flooding in Nuclear Power Plant Siting in Coastal Area. (GHME20130193).

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