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# Incremental Loading Balance Method for the Widening and Reinforcement Design of Old Beam Bridges

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#### Abstract

To increase utilization of the old beam bridges with narrow deck and inadequate bearing capacity, the incremental loading balance method (ILBM) was proposed based on theoretical analysis and the widening and reinforcement method with longitudinal and transverse beams. Calculation analysis models of old beam bridges before and after reconstruction were established based on the fundamental principle of elastic mechanics and the principle of generalized force equilibrium, respectively. From the comparison of these two models, the method of calculating and optimizing sectional dimensions of new longitudinal and traverse beams was developed. Finally, the reliability and validity of the proposed method were verified using a 20 m-span simply-supported prestressed concrete girder bridge which is located in Yueyang City, Hunan Province of China. Results show that the ILBM could determine the size of new longitudinal and traverse beams is determined by the incremental bearing capacity of the entire bridge, whereas that of traverse beams is determined by the incremental bearing capacity of the single beam. The widths of the longitudinal and traverse beams increase with the increase in their stiffness, and the heights decrease with the increase in bending moment of the old bridge. The study can meet the demand of the reconstruction of old bridges.

Keywords: Bridge Engineering, Old Bridges, Widening and Reinforcement Design, Increment Load Balance Method

#### 1. Introduction

With the high-speed economic development in China, the traffic and bearing capacities of bridges should be increased to meet the continuous growth of the volume of traffic and heavy-duty vehicles. Thus, the departments in the traffic industry increase the design loading level of bridges. However, the old bridges constructed according to the old standards present potential safety risks. Combined with structural component cracking [1], material aging [2], and structural damages in operation, some old beam bridges could not satisfy the load requirements [3]. Moreover, some old bridges with narrow decks could not meet the usage requirement as the volume of traffic increases. Dismantling and reconstruction of old bridges not only waste construction resources and influence the ecological environment, but also cause long-term traffic interruption and poor social influence. Therefore, widening and reinforcement of old bridges are the best approaches to solve these problems. How to find a feasible and effective bridge widening and reinforcement method is a problem that needs to be solved urgently in the traffic research field.

The widening and reinforcement method with longitudinal and traverse beams was put forward in Reference [4]. This widening and reinforcement method is a widening-based reinforcement method for the reconstruction of old bridges. How to determine stiffness of longitudinal and traverse beams for the widening and reinforcement of

\* E-mail address: wgh325@126.com ISSN: 1791-2377 © 2016 Eastern Macedonia and Thrace Institute of Technology. All rights reserved. old bridges was discussed theoretically in this paper. The loading features of single-beam and multi-beam bridges before and after widening and reinforcement were discussed based on the fundamental principle of elastic mechanics. The corresponding calculation analysis models were established. Thus, the incremental loading balance method (ILBM) for the widening and reinforcement of old beam bridges was proposed based on the principle of generalized force equilibrium.

#### 2 State of the Art

Existing researches concerning the utilization of old bridges mainly focused on either widening or reinforcement. Only a few studies discussed widening and reinforcement together, and even fewer associated theoretical works are available. For the reinforcement of old bridges, J. Nilimaa et al. reinforced one idle bridge by CFRP and tested the reinforcement effect through a destructive test [5]. T. Guo et al. investigated the time-dependent reliability of paste steel plate, paste FRP, and external prestressing in reinforcing a box girder bridge [6]. O. Chaallal et al. conducted antibending and shearing tests on a concrete beam with reinforced fiber plastic plates [7]. These studies focused on incremental bearing capacity related with different reinforcement materials and different reinforcement methods, which only increased the bearing capacity in a limited range. With respect to widening old bridges, Y.H. Chai et al. explored the time-dependent characteristics of differential settlements between new and old girders of the widened

bridge [8]. T Jirawattanasomkul et al. conducted an experimental study on the shear behavior of reinforced concrete members fully wrapped with polyethylene terephthalate fiber-reinforced polymer composites [9]. Wu Wenging et al. conducted an experimental study on the horizontal distribution pattern of settlement deformation of new plates used in a wide hollow slab bridge [10]. These studies did not involve the influences of widening on the bearing capacity of bridges. Only a few studies combined widening and reinforcement, particularly widening and reinforcement with longitudinal and traverse beams. The widening and reinforcement method with longitudinal and traverse beams [4] involves the construction of one large marginal girder on two sides of the old bridge. These large marginal girders are longitudinal beams. The internal flange plates of the large marginal girders are connected with the external flange plates of two side beams of the bridge. One traverse beam supporting the old girder is constructed at the midspan, with two ends bonded to the large marginal girders. These new longitudinal and traverse beams form an "Hshaped" structure, in which the large marginal girders achieve widening and the new traverse beam supporting the old girder ensures reinforcement. Numerous studies have investigated large marginal girders. Li Guohao and Shi Dong proposed a correction method to load the transverse distribution of rigid-jointed girder bridge with large marginal girders by calculating the adjustment value of transverse distribution through the fundamental equation of force method based on equal rigidity [11]. Wu Qingxiong et al. described the proportional relations between rigidity of the middle beam and deflections, bending moments, shearing forces, and loads. They compiled a calculation program of the load transverse distribution factor of bridges with large marginal girders based on hinge-jointed plate method and rigid-jointed beam method [12]. Zhang Yuanhai and Li Qiao viewed the extra rigidity of the large marginal girder compared with the rigidity of the middle beam as one virtual longitudinal beam and deduced the corresponding calculation formula. The researchers indicated that the stress of the old girder could not be improved after the rigidity of large marginal girders reaches a certain value [13]. Bao Longsheng and Yu Ling et al. conducted a scale model test on the old bridge reinforcement with large marginal girders and designed a formula for the load transverse distribution factor involving stiffness correction. However, this formula showed poor accuracy [14]. These studies neglected the situation in which a traverse beam was used. Wang Guanghui et al. proposed the concept of widening and reinforcement of old bridges with longitudinal and traverse beams for the first time [4]. Wang Guanghui and Wei Chenglong determined the stiffness of longitudinal and traverse beams for a reinforced T-beam bridge using the parameter analysis method and employed the results in an actual bridge, resulting in an excessively large structure and material wastes [15]. Liu Xiaoyan and Chen Wei et al. built several calculation models of widening and reinforcement of a hollow slab bridge with longitudinal and traverse beams. The stiffness of longitudinal and traverse beams was determined via the parameter analysis method [16].

To sum up, previous research on the widening and reinforcement of old bridges has mainly focused on practical engineering. Studies on the calculation theory of widening and reinforcement emphasize the calculation methods of the internal force of old bridges reinforced with large marginal girders. However, bridges reconstructed by widening and reinforcement with longitudinal and traverse beams possess one more new traverse beam at the midspan compared with bridges reconstructed with large marginal girders. The two reconstruction methods possess different force transfer paths. This study investigated the stiffness calculation of new longitudinal and transverse beams in the widening and reinforcement method with longitudinal and traverse beams. The ILBM was proposed to ensure the safety and quality of the reconstructed bridges and increase the calculation efficiency.

This study is organized as follows: Section 3 introduces the calculation hypothesis, establishes the calculation models of old bridges before and after reconstruction, and proposes the ILBM. Section 4 verifies the reliability and practicability of the proposed method through a widening and reinforcement design for a simply-supported prestressed concrete girder bridge. Section 5 presents the conclusions.

# 3. Methodology

To elaborate the ILBM for the widening and reinforcement design of old bridges, the stress conditions of single-beam and multi-beam bridges before and after the reconstruction were analyzed based on the following hypothesis.

# 3.1 Calculation Hypothesis

A bridge reconstructed by widening and reinforcement with longitudinal and traverse beams is a composite structure. For the convenience of calculation, structure size changes and influences of longitudinal and traverse beams on the transverse load distribution of the old girder after reinforcement are neglected. Zhang et al. indicated that new longitudinal beams would decrease the transverse load distribution factor of the old girder after widening [13]. In this hypothesis, the transverse load distribution factor of the old girder remains unchanged before and after the reconstruction.

# 3.2 Single-beam Bridge

The nonqualified single-beam bridge is shown in Fig. 1. The widening and reinforcement method with longitudinal and traverse beams is used to reinforce this single-beam bridge.

# **3.2.1** Calculation

The calculation of the old single-beam bridge is shown in Fig. 1(b). The design load of the bridge is P, and the maximum bearing capacity is  $P_1$ . If  $P_1 < P$ , then the bridge possesses inadequate bearing capacity. The widening and reinforcement method with longitudinal and traverse beams is applied to increase the bearing capacity. The calculation diagram of the bridge after reinforcement is shown in Fig. 1(c), where AB is the girder of the original old bridge,  $A_1B_1$  and  $A_2B_2$  at two sides of the old bridge are new longitudinal beams (the large marginal girders), and  $C_1C_2$  is the new transverse beam that supports the old girder. The top surface of  $C_1C_2$  is in good contact with the bottom surface of AB.



Fig.1. Single-beam bridge diagram

For protecting structural safety, the new longitudinal and traverse beams are responsible for the incremental loading ( $\Delta P$ ):

$$\Delta P = P - P_1,\tag{1}$$

where *P* is the maximum load of the bridge during the running period, that is, the design load;  $P_1$  is the maximum bearing capacity of the bridge;  $\Delta P$  is the load on the reinforcement part, that is, incremental loading; and *P*,  $P_1$ , and  $\Delta P$  are all generalized forces, which generally refer to bending moment, shearing force, and axial force, respectively.

#### **3.2.2 Determination of** $\Delta P$

In Fig. 2(a), the deflection of C cross-section at the midspan of the old girder under the effect of  $P_1$  is  $f^0$ . The stress conditions of the old girder under the design load after

reconstruction are shown in Fig. 2(b). According to Equation (1), Figs. 2(b) and 2(c) are consistent and  $f^0 = f$ .

For the widening of old bridges, a downward load ( $\Delta P$ ) is applied to the traverse beam. Under this circumstance, the deflection at point *C* is f'. Apparently,  $f' = f < f^0$ , indicating that the reinforcement can meet the requirement.

## 3.3 Multi-beam Bridge

Fig. 3(a) shows a multi-beam bridge. The old bridge can be simplified into Fig. 3(c) by calculating the load transverse distribution factors of different girders. The maximum bearing capacity and the design load of beam *i* are  $P_{1i}$  and  $P^i$ , respectively. The bridge with  $P_{1i} < P^i$  at any beam is

evaluated to be poor in terms of bearing capacity and must be reinforced.





Fig. 4. Calculation diagram of a multi-beam bridge

### 3.3.1 Calculation

The calculation diagram of the bridge after widening and reinforcement is shown in Fig. 4(a).  $A_iB_i$  (i = 1, 2, ..., n, n is the number of old girders) represents longitudinal beams at two sides.  $A_{11}B_{11}$ ,  $A_{22}B_{22}$ , and  $C_1C_2$  form an "H-shaped" structure. The top surface of  $C_1C_2$  is in good contact with the bottom of the old beam  $A_iB_i$ .

For protecting structural safety, the new "H-shaped" structure must bear the incremental loading for beam i  $(\Delta P_i)$ :

$$\left\{\Delta P_i\right\} = \left\{P^i\right\} - \left\{P_{1i}\right\},\tag{2}$$

where  $P^i$  is the maximum load of beam *i* during the running period, that is, design load;  $P_{1i}$  is the maximum bearing capacity of beam *i* after the emergency capacity is considered;  $\Delta P_i$  is the load on beam *i* in the reinforcement part, that is, incremental loading; And  $P^i$ ,  $P_{1i}$ , and  $\Delta P_i$  are

all generalized forces, which generally refer to bending moment, shearing force, and axial force, respectively.

## **3.3.2 Determination of** $\Delta P_i$

In Fig. 5(a), the maximum bearing capacity of any old girder (such as beam *i*) is applied to the midspan cross-section  $C_i$ . At this moment, the deflection of  $C_i$  is  $f_i$ . According to Equation (2), the structure shown in Fig. 5(a) is consistent with that in Fig. 4(b). An upward load  $\Delta P_i$  is applied at point  $C_i$ , which is the support force of the new H-shaped structure to beam *i*. For the H-shaped structure, a downward load  $\Delta P_i$  is applied at the corresponding position of the traverse beam (Fig. 5(b)). The deflection at this point under the effect of  $\Delta P_i$  is  $f_i'$ . In Fig. 5(c), the reinforcement of beam *i* is satisfactory as long as  $f_i' \leq f_i$ . When i = 1, 2, ..., n (*n* is the number of old girders), the bridge reinforcement can meet the usage requirements as long as all beams possess  $f_i' \leq f_i$ .



Fig. 5. Calculation mode of a multi-beam bridge

# 3.4 Determination of the Reasonable Stiffness of Longitudinal and Transverse Beams

The reasonable stiffness of longitudinal and transverse beams could be determined according to the following steps: (1) Calculate the maximum bearing capacity array  $\{P_{li}\}$  of the girder and the corresponding deflection array  $\{f_i\}$ .

When the ultimate load is expressed as the ultimate

bending moment  $M_{u}^{i}$ ,

$$P_{1i} = M_{u}^{i} = f_{cd}^{i} b_{i} x_{i} (h_{0}^{i} - \frac{x_{i}}{2}), \qquad (3)$$

where  $f_{ad}^{i}$  is the designed axial compressive strength of concrete in beam *i*;  $x_i$  is the height of the concretecompressed region in the calculation cross-section of beam *i*;  $b_i$  is the effective width of the calculation cross-section of beam *i*; and  $h_{o}^{i}$  is the effective height of the calculation cross-section of beam *i*. For the beam with degraded material properties,  $h_{o}^{i}$  and  $b_{i}$  are the cross-section sizes after material degradation is considered.

The deflection  $\{f_i\}$  can be expressed as:

$$f_i = \alpha \frac{M_u^i l_i^2}{E_c^i l_0^i},\tag{4}$$

where  $\alpha$  is a coefficient related to the shape and boundary conditions of the bending moment diagram,  $M_u^i$  is the ultimate bending moment of the calculation cross-section of beam *i*;  $l_i$  is the calculated span of beam *i*;  $E_c^i$  is the elasticity modulus of concrete of beam *i*; and  $I_0^i$  is the conversion moment of inertia of beam *i* after material degradation and structure damage are considered.



Fig. 6. Flowchart of the ILBM

(2) Determine the design load array  $(\{P_i\})$  under the "new" class of loading.

$$P^{i} = (1+\mu)\mathcal{E}(m_{i}py + \sum m_{i}q\Omega), \qquad (5)$$

where  $\mu$  is the impact coefficient,  $\xi$  is the reduction

coefficient,  $m_i$  is the transverse distribution factor of beam i, p is the concentrated force of the bridge lane loading, q is the uniform load of the bridge lane loading, y is the longitudinal coordinate value of the influence line corresponding to p, and  $\Omega$  is the area of the influence line of beam i.

(3) Calculate the incremental loading balance array  $\{\Delta P_i\}$ and deflection array  $\{f_i'\}$ .

(4) Judge. If  $\{f_i^{'}\} \leq \{f_i\}$  (i = 1, 2, ..., n), then the reinforcement design is satisfied; otherwise, a new reinforcement design is needed. In addition, whether the bridge is over-reinforced is determined according to the degree of min $\{f_i / f_i^{'}\} > 1$ .

The flowchart to determine the stiffness of new longitudinal and transverse beams in the reconstruction of old bridges is shown in Fig. 6.

### 4 Result Analysis and Discussion

The widening and reinforcement method with longitudinal and transverse beams has to determine the incremental bearing capacity of old bridges and the size of the H-shaped structure. The size of longitudinal beams is determined according to the actual bearing capacity and design load of the superstructure of the old bridge as well as the self-weight of the longitudinal and transverse beams, whereas the size of the transverse beam should be determined according to the actual bearing capacity of each single beam. In practical engineering, the height and width of the transverse beam are determined first. The height of longitudinal beams is the sum of the heights of the old girder and the transverse beam. The width of the longitudinal girder rib is determined according to the incremental bearing capacity. In this study, the reliability and calculation efficiency of the ILBM are verified by a  $2 \times 20$  m simply-supported prestressed concrete girder bridge on the 021 Country Road in Yueyang City, Hunan Province.

In the widening and reinforcement design, large marginal girders are constructed at two sides of every span in the old bridge and a new transverse beam at midspan, forming an H-shaped structure. Steel bars are embedded in old beams above the transverse beam, which are integrally welded into reinforcing steel bars in the transverse beam. The cross-section after reconstruction is shown in Fig. 7.



Fig. 7. Cross-section of the bridge after reconstruction

# 4.1 Determination of the Reasonable Aspect Ratio of the Transverse Beam

The sizes of the longitudinal and transverse beams should be determined preliminarily in the calculation. The larger the

cross-section of the transverse beam is, the smaller the deflection under the effect of  $\Delta P_i$  is. Therefore, influences of the aspect ratio of the transverse beam on bridge reconstruction are analyzed under the fixed cross-section size of the transverse beam.

<b>TADIC 1.</b> CT055-Section Size of the transverse beam
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Height of the transverse beam (cm)	42.43	50	60	70	80	
Width of the transverse beam (cm)	42.43	36	30	25.7	22.5	
Moment of inertia $I (\times 10^{-8} \text{ m}^4)$	264,600	375,000	540,000	735,000	960,000	



Fig. 8. Relation curve between the internal force of the old girder and the beam height

In Fig. 8, the bending moments of the marginal beams and middle beam of the old bridge decrease as the height of the new transverse beam increases. The bending moment variation curve of the middle beam intersects with that of the marginal beam and extends downward continuously. Two curves intersect when the beam height is 53 cm. The beam height is determined to be 53 cm. In practical engineering, the proposed beam width is 30 cm considering the layout of reinforced steel bars and the size of the reinforcement cover in the transverse beam. If the aspect ratio of the transverse beam is generally 1:2 and the transverse beam presents a certain emergency capacity, the initial height of the transverse beam is determined to be 60 cm. Given that the contribution of the transverse beam height to stiffness is the cube of its width, the aspect ratio should be decreased from 1:2 to identify the reasonable size.

#### 4.2 Comparison of Two Methods

In Reference [14], 36 finite element models of the engineering example were constructed to analyze the stiffness of longitudinal and transverse beams. According to the relation curve of stiffness of new longitudinal and transverse beams with internal force and deflection of the old girder, the height of new longitudinal beams and the cross-section size of the transverse beam were determined to be 2.5 m and 60 cm  $\times$  30 cm, respectively.

According to the proposed ILBM, the cross-section size of the transverse beam and height of longitudinal beams are preliminarily determined to be 60 cm  $\times$  30 cm and 1.5 m + 0.6 m = 2.1 m, respectively. According to the calculations shown in Fig. 6, the height of the longitudinal beams, the width of the beam rib, and the cross-section size of the transverse beam are finally determined to be 2.1 m, 48 cm, and 60 cm  $\times$  30 cm, respectively. In this case, min { $f_i / f'_i$ } = 1.05, indicating that the calculated H-shaped

structure is reasonable in size and the old bridge is not overreinforced.

Although the sizes of the longitudinal and transverse beams determined by parameter analysis could meet thesafety requirements, the cross-section is excessively large. The ILBM could determine the reasonable crosssection sizes of the longitudinal and transverse beams according to the actual necessary incremental bearing capacity of the structure. Moreover, the approach requires a small computation load and can obtain the ideal results by establishing several models.

#### **5** Conclusions

This study investigated the widening and reinforcement method with longitudinal and transverse beams to increase the utilization of old bridges with narrow deck and inadequate bearing capacity. The reasonable stiffness of the new longitudinal and transverse beams was determined first. Then, the stress of the composite structure comprising large marginal girders, the transverse beam, and the old bridge was analyzed. From the fundamental principle of elastic mechanics and combined with the principle of generalized force equilibrium, the calculation analysis models for widening and reinforcement of single-beam and multi-beam bridges are constructed. The following conclusions are drawn:

(1) The ILBM for the widening and reinforcement design of old bridges is proposed. The method determines the stiffness of longitudinal and transverse beams according to the incremental loading  $\Delta P_i$  and judges whether the sizes of the longitudinal and transverse beams have to be optimized according to min{ $f_i / f_i$ }.

(2) The size of the new transverse beam is determined by the incremental bearing capacity of the single beam. The size of the new transverse beam is determined by the actual bearing capacity of each single beam. In the actual construction, the proposed width of the transverse beam is 30 cm. The stiffness of the new longitudinal beams is determined by the incremental bearing capacity of the bridge. The initial height of longitudinal beams is the sum of the heights of the older girder and the transverse beam.

(3) The ILBM is proven to be reliable and have a small computation load.

The ILBM is proposed by combining engineering practices and theoretical studies, and the method has practical significance. The bridge that is reconstructed by the longitudinal and transverse beams is a composite structure. For convenience of calculation, a hypothesis is proposed: the load transverse distribution factor of the bridge remains constant before and after reconstruction. Therefore, further research on the composite structure model and the ILBM that involves changes in the load transverse distribution factor is needed.

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